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Journal
Journal of International Economics, 123(7500)

ISSN
0022-1996

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Publication Date
2020-03-01

DOI
10.1016/j.jinteco.2020.103295

Peer reviewed

Arming in the Global Economy: The Importance of Trade with Enemies and Friends

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Current Version: January 15, 2020

Abstract: We analyze how trade openness matters for interstate conflict over productive resources. Our analysis features a terms-of-trade channel that makes security policies trade-regime dependent. Specifically, trade between two adversaries reduces each one's incentive to arm given the opponent's arming. If these countries have a sufficiently similar mix of initial resource endowments, greater trade openness brings with it a reduction in resources diverted to conflict and thus wasted, as well as the familiar gains from trade. Although a move to trade can otherwise induce greater arming by one country and thus need not be welfare improving for both, aggregate arming falls. By contrast, when the two adversaries do not trade with each other but instead trade with a third (friendly) country, a move from autarky to trade intensifies conflict between the two adversaries, inducing greater arming. With data from the years surrounding the end of the Cold War, we exploit the contrasting implications of trade costs between enemies versus trade costs between friends to provide some suggestive evidence in support of the theory.

JEL Classification: D30, D74, F10, F51, F52.
Keywords: resource insecurity, interstate disputes, conflict, arming, trade openness, comparative advantage

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1 Introduction

International trade takes place within an anarchic setting. In the absence of an ultimate adjudicator and enforcer, countries inevitably have unresolved disputes and nearly all expend resources on defense to prepare for the possibility of outright conflict or to improve their bargaining positions under the threat of conflict. Despite the anarchic nature of international relations, the classical liberal perspective views greater trade openness among potential adversaries as reducing or even eliminating conflict (e.g., Polachek, 1980). One argument in support of this view builds on the disruptive nature of conflict that prevents the realization of at least some of the gains from trade; then, countries acting collectively and wanting to reap those gains would have a greater interest under trade to maintain a peaceful order. However, as is well-known with the prisoners' dilemma being a stark and simple example, collective rationality need not and often does not prevail. Indeed, emphasizing self-interest and individual rationality as well as the anarchic nature of international relations, the realist/neo-realist perspective argues, in contrast to the classical liberal view, that trade can aggravate conflict between nations (e.g., Waltz, 1979). Specifically, the benefits from freer trade can fuel frictions, as some states perceive that they (or their rivals) will gain a military edge in security competition.[1]

Our central objective in this paper is to study how the expansion of international trade affects the intensity of conflict, measured in terms of resources allocated to it. Consider, for example, the ongoing dispute involving China, Taiwan, Vietnam, the Philippines, Indonesia, Malaysia, and Brunei for control over the Spratly and Paracel islands in the South China Sea, where there are oil reserves. Does trade between these rivals pacify their relations inducing them to allocate fewer resources to their respective militaries as might be implied by the classical liberal view, or does it make their rivalry more severe?[2]

Previous theoretical treatments of trade and conflict in the economics literature have identified two distinct channels through which trade between countries could influence their military spending: a factor-price channel and an income channel. For example, based on extensions of Heckscher-Ohlin models that emphasize differences in factor endowments, Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015) study settings with two small

[1] Pushing this logic one step further, one could argue that actual or potential rivals would not trade with each other (e.g., Grieco, 1990; Gowa, 1995). See Copeland (2015) for a recent survey of the theoretical and empirical literature in international relations regarding trade and conflict.

[2] Similarly, one might ask how globalization in the past few decades has influenced tensions between Russia, Kazakhstan, Turkmenistan, Iran, and Azerbaijan over how to divide the rights of exploration and exploitation for oil in the Caspian Sea. Although these countries came to an agreement in August 2018, many of the details have yet to be worked out, leaving open the possibility that this dispute could continue for some time. (See https://www.nytimes.com/2018/08/12/world/europe/caspian-sea-russia-iran.html and https://www.bbc.com/news/world-4516228..) It is worth adding here that historically trade has occurred between countries even while they were at war with each other—e.g., Standard Oil selling oil to Nazi Germany. (See Barbieri and Levy (1999) and references cited therein for additional examples.)

countries that contest a resource and can trade in world markets. A shift from autarky to trade in such settings changes product prices and thus relative factor prices, and thereby alters the cost of combining resources to produce military force. Depending on world prices, then, trade can either amplify or diminish the countries' incentives to arm and, as a result, either intensify or pacify their dispute over the resource. Garfinkel et al. (2019) abstract from this channel to highlight the income channel in a dynamic model, where two countries make consumption, investment and arming decisions in the first period, as they face a strictly positive probability in the next period of having to contest a portion of the combined returns from their first-period investments. The authors find that trade in the first period raises the incomes of both countries and their production of arms, with the initially smaller country gaining some power relative to its larger trading partner and potential foe.

The analysis in this paper, based on a variant of the Ricardian model suitably extended to allow for international disputes, studies a third channel through which trade can matter for military spending—namely, a terms-of-trade (TOT) channel. As in the canonical, two-good, two-country Ricardian model, international differences in technology serve as the basis for comparative advantage and provide the rationale for mutually advantageous trade. However, in a departure from that model, we assume the input to the production of potentially traded goods is produced with two primary resources, one of which is partially insecure and thus subject to dispute. The disputed resource could be oil, minerals, timber, land, or water.[3] Its division depends on the countries' "arms" or "guns," also produced domestically using the two primary resources. Since arming is endogenous, so too are the residual resources and the intermediate input used in the production of consumption goods.

A key feature of this setting is the endogeneity of world prices that makes security policies trade-regime dependent.[4] To highlight the importance of this mechanism, we construct a simple model that abstracts from the factor-price and income channels mentioned above.[5] Furthermore, we abstract from many salient features of today's world economy, such as the presence of increasing returns, foreign investment, and growth. In addition, we do not differentiate between the mobilization of resources for conflict and the potentially destructive deployment of those resources, nor do we consider explicitly the disruptive effect of conflict to

[3]See Klare (2012), who provides many examples where the competition for scarce resources, for which property rights are not well defined or costlessly enforceable, has turned or can turn violent.

[4]Insofar as trade policies can influence world prices as suggested by the empirical work of Broda, et al. (2008) and Bagwell and Staiger (2011), world prices should depend on resources available to produce traded goods and thus on arming as well. It stands to reason, then, that policymakers take that influence into account when choosing their security (and possibly trade) policies.

[5]Likewise, in the small country settings considered by Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015), the TOT channel of influence is non-operative. While world prices are endogenous in Garfinkel et al. (2019), they are determined independently of the countries' arming choices. Of course, in more general settings, all three channels of influence could be present.

shut down trade between warring nations.[6] Instead, motivated by the empirical relevance of military expenditures, we focus on arming.[7] Insofar as resources are absorbed into the production of arms (thus becoming unavailable for producing goods traded in world markets), these expenditures represent an additional opportunity cost of conflict. Given our focus, this paper can be thought of as an analysis of a modified version of the classical liberal view, one that applies to "cold" wars.

We compare the outcomes under two polar trade regimes, autarky and free trade. As one would expect, given the amount of resources allocated to arming, a shift from autarky to free trade unambiguously results in higher payoffs to both countries. However, because in our setting such a switch also influences arming decisions (or security policies), the welfare consequences of introducing or liberalizing trade can differ significantly from those that typically arise in mainstream analyses, which assume perfect and costless security.[8]

Under autarky, each country chooses its security policy so as to equate the marginal benefit of capturing the contested resource to the marginal cost of diverting resources from its own production and thus consumption. At the same time, each country's arming choice adversely affects the opponent by reducing its access to the contested resource. In equilibrium, both countries ignore this negative security externality and arming is strictly positive.

Importantly, trade induces each country to internalize, at least partially, the negative externality of its security policy on the resources available to its rival. The result is a lower incentive to arm given the rival's policy. To be more precise, as in the case of autarky, when the two countries trade with each other, each one chooses its arming to balance its marginal benefit with its marginal cost. In the case of trade, however, each country's payoff depends on the production of its adversary's exported good. Accordingly, an increase in one's own arms has an additional cost under trade: a reduction in the adversary's share of the contested resource and thus a reduction in the adversary's production of its exports. The added cost, which is reflected in a deterioration of the importing country's TOT that lowers the overall marginal benefit of arming relative to the marginal cost, means that a country's incentive to arm, given the adversary's arming choice, is strictly lower under free

[6]This is not to deny the importance of conflict's disruptive effect on trade. To the contrary, this effect is empirically relevant (Glick and Taylor, 2010), and can be viewed as an opportunity cost of conflict that serves as the basis for the classical liberal view as suggested above. In related research, Garfinkel and Syropoulos (2019) explore the adversarial countries' arming choices and their subsequent decision to either fight that would preclude trade between them or negotiate a peaceful settlement.

[7]Researchers from the Stockholm International Peace Research Institute (SIPRI) estimate that, in 2018, global military expenditures were \$1,822 billion, accounting for 2.1 percent of global GDP. The five biggest spenders that year (in current U.S. dollar terms) were the United States (\$649 billion or 3.2 percent of GDP), China (\$250 billion or 1.9 percent), Saudi Arabia (\$67.6 billion or 8.8 percent), India (\$66.5 billion or 2.4 percent), and France (\$63.8 billion or 2.3 percent). See Tian et al. (2019) for more details.

[8]Nevertheless, our welfare analysis and comparison of trade regimes resemble those in Arkolakis et al. (2012) and Costinot and Rodríguez-Clare (2014). The primary difference is that, in our work, real income is endogenously determined under non-cooperative interactions in arming.

trade than under autarky. This finding is robust to the presence of trade costs and does not disappear when countries simultaneously and non-cooperatively choose tariffs.

Of course, the effect of a shift from autarky to trade on *equilibrium* arming and thus on welfare depends not only on the direct effect given the rival's arming choice, but also on the indirect or strategic effect as the rival country's arming changes. However, we find that the strategic effect is, by and large, of second-order importance. That is to say, provided the distribution of the contested resource (what we call "capital") and the uncontested resource (what we call "labor") across the adversarial countries is not severely uneven, the strategic effect tends to reinforce the direct effect or, even if it moves in the opposite direction, is swamped by the direct effect. In either case, since trade is no worse than autarky for a given level of arming, the reduction in arming and thus security costs render trade unambiguously superior to autarky. This added benefit, not captured by mainstream trade theory that abstracts from the insecurity of property, is consistent with the spirit of classical liberalism and the writings of authors such as Angell (1933) who extolled the virtues of trade openness and globalization.

But, there do exist sufficiently asymmetric distributions of resources where one country has a sharply larger ratio of capital to labor than its adversary to imply a sharply greater opportunity cost of arming; this country arms by less than its rival under both trade regimes, but a shift to trade induces it to increase its arming as its rival reduces its arming.[9] Even though the country that is induced to arm by less remains more powerful, the resulting adverse strategic effect it realizes could swamp its gains from trade to render trade unappealing. This finding is reminiscent of the realist/neorealist view in the international relations literature (mentioned above) that highlights trade's effect to generate uneven gains to trading partners and thereby differentially influence their arming and thus the balance of power.[10] However, we find that the differential influence of trade on the two countries' arming choices and the implied influence on the balance of power hinge on sharp differences in the mix of their initial holdings of capital and labor, not simply on differences in the size of their economies.[11] Nonetheless, even in such cases, we find that a move to trade, whether costly or not, results in lower aggregate arming.[12]

[9]This possibility is consistent with the empirical finding of Morelli and Sonno (2017) that asymmetries in oil endowments across two countries reduce the pacifying effect of bilateral dependence between them in trade and with Beviá and Corchón's (2010) theoretical finding that sharp asymmetries in endowments can reduce the effectiveness of resource transfers between countries to avoid war.

[10]Also see Garfinkel et al. (2019) mentioned above.

[11]Also see Bonfatti and O'Rourke (2018), who consider the role of increasing trade dependence for two adversarial countries with the rest of the world, in a dynamic, leader-follower setting, to influence the likelihood of a preemptive war by the follower. In that analysis, similar to ours, understanding the emergence of conflict between the two countries does not hinge on differences in the size of their economies. But, in Bonfatti and O'Rourke's analysis, one fundamental sort of asymmetry is critical—that is, the leader's ability to block imports (necessary for arming) to the rival.

[12]Acemoglu and Yared (2010) find empirically that military expenditures and the size of the military in

The logic of the TOT channel has sharply different implications for the effect of increased integration of world markets on the incentives to arm by adversarial countries that do not directly trade with each other, but instead with a friendly country. To isolate these differences, we focus on adversaries having a similar comparative advantage.[13] Suppose, in particular, that the two adversarial countries are identical in every respect so that, even in the absence of barriers to trade, they would not trade with each other. Also, suppose that there exist technological differences between these countries on the one hand and the third country on the other hand that, given the resources allocated to arming, make trade mutually advantageous. Since the two adversaries compete in the market for the same good exported to a third country in this case, arming generates an added marginal benefit under trade through the TOT channel to increase arming incentives under trade relative to those under autarky. That expanded trade opportunities with another (friendly) country can intensify conflict between two adversaries is similar in essence to Martin et al. (2008), who show that increasing the opportunities for trade among all countries reduces the interdependence between any two and thus can make conflict between them more likely. However, while that analysis emphasizes the importance of the disruptive effects of conflict, ours underscores the importance of the endogeneity of arming and its trade-regime dependence. Furthermore, our analysis, like Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015), suggests that, in this case, trade between friends could make the adversaries worse off.[14]

Based on the different implications of trade with the enemy versus trade with friends for arming choices, we also provide some suggestive evidence in support of the theory. Our empirical analysis studies how trade openness, measured inversely by trade costs, influences national military spending, with data surrounding the end of the Cold War. An essential feature of this analysis lies in the distinction made, using data on bilateral "strategic rivalries" from Thompson (2001), between countries that have no rivals and countries that do have rivals. For countries having rivals, we further differentiate between their trade costs with rivals and their trade costs with friends. This distinction is statistically significant, with qualitative differences as predicted. Specifically, the estimates confirm that a country's military spending is positively related to its trade costs with rivals, but inversely related

terms of personnel are negatively related to trade volumes. Although that analysis treats military variables (reflecting a country's "nationalist" or "militarist" sentiments) as exogenous, the negative relationship found can be viewed as preliminary evidence in support of the modified version of the classical liberal view we consider here, focusing on military expenditures. In any case, as discussed below, we provide additional evidence in support of this view.

[13]Consider, for example, India and Pakistan and their ongoing conflict over Kashmir.

[14]It is important to emphasize, though, that the result in this paper (with trade between friends) is due to the beneficial effect that a country's arming has on its own TOT to add to arming incentives, whereas the result in Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015) derives from the impact of world prices to increase arming incentives through factor prices. A similar result arises in settings with conflict over some resource between groups within a single (small) country that trades with the rest of the world (e.g., Garfinkel et al., 2008).

to its trade costs with friends. These results, which are robust to a series of alternative specifications that address concerns of possible endogeneity of the trade-cost variables in the econometric model, complement the more structural evidence provided by Martin et al. (2008) that increased trade flows need not always promote peace and by Seitz et al. (2015) that reductions in trade costs bring added benefits largely through their effect to dampen defense spending, not only by trading partners, but other countries too.

In what follows, the next section presents the basic model, focusing on just two countries that trade with one another. In Section 3, we characterize the countries' incentives to arm under each trade regime. Section 4 studies how equilibrium arming and payoffs compare across trade regimes and discusses how the central results remain intact with the introduction of trade costs. In Section 5, we extend the framework by introducing a third (non-adversarial) country. In Section 6, we present our empirical evidence. Concluding remarks follow in Section 7. Technical and supplementary details are relegated to appendices.

2 Conflict in a Modified Ricardian Setting

Consider a world with two countries, indexed by superscript $i = 1, 2$. Each country i holds secure endowments of two productive resources: labor, denoted by L^i, and capital denoted by K^i. Capital could be land, oil, minerals, timber or water resources. In contrast to standard trade models, we suppose there is an additional amount of productive capital, denoted by K_0, that is not held securely by either country and is subject to dispute.[15] In particular, each country i can use a fraction of its secure holdings of labor and capital to produce (via a constant returns to scale or CRS technology) a composite good, "guns," denoted by G^i. Each country's guns reflect its military strength used to contest K_0. Once property rights over K_0 are established, each country i employs its available resources to produce (again via a CRS technology) an intermediate good, Z^i. In turn, this intermediate good serves as the unique input to the production of two (and potentially tradable) final consumption goods. As in the canonical Ricardian model, markets are perfectly competitive, and comparative advantage is due to international differences in technology.

We present our model in three steps. First, we describe the baseline model of trade, *for given guns* G^i and thus for a given resource base available to produce the intermediate good Z^i. Second, departing from the canonical Ricardian model of trade, we introduce conflict, and show how guns affect this resource base and thus the production of Z^i. Finally, we describe the determination of equilibrium for given guns and derive the associated payoff functions that play a central role in our subsequent analysis of endogenous guns choices, contingent on the trade regime in place.

[15] With an appropriate choice of K_0 and K^i, our analysis could capture conflicts over what would appear to be one country's resource. See Caselli et al. (2015), who examine empirically the importance of the proximity of oil fields to countries' shared border in the escalation of conflict between them.

2.1 Baseline Model of Trade for Given Guns

Preferences and demand functions. As in the standard Ricardian model, consumer preferences in each country $i = 1, 2$ are defined over the two consumption goods and captured by $U^i = U(D^i_i, D^i_j)$, where D^i_j denotes the quantity of good j $(= 1, 2)$ a representative consumer in country i demands.[16] This utility function is increasing, quasi-concave, homogeneous of degree one and symmetric over the two goods, with a constant and finite elasticity of substitution, denoted in absolute terms by $\sigma \in (0, \infty)$.[17]

Let p^i_j and Y^i denote respectively the price of consumption good j and national income in country i. Then, country i's indirect utility function can be written as $V^i = \mu^i Y^i$, where $\mu^i = [(p^i_i)^{1-\sigma} + (p^i_j)^{1-\sigma}]^{1/(\sigma-1)}$ (for $i \neq j = 1, 2$) represents the marginal utility of income, which is decreasing and homogeneous of degree -1 in prices.[18] By Roy's identity, the demand function for good j in country i is

$$D^i_j = \gamma^i_j Y^i / p^i_j, \tag{1}$$

where $\gamma^i_j \equiv \gamma^i_j(p^i_i, p^i_j) = -(\partial\mu^i/\partial p^i_j)/(\mu^i/p^i_j) = (p^i_j)^{1-\sigma}/[(p^i_i)^{1-\sigma} + (p^i_j)^{1-\sigma}]$ denotes country i's expenditure share on good j.

Production possibilities of final goods. Given the quantity of intermediate good Z^i in country i $(= 1, 2)$, let S^i_j denote the quantity of its final good j $(= 1, 2)$. To produce one unit of good j, producers in country i need $a^i_j > 0$ units of Z^i. Thus, each country $i = 1, 2$ faces the following production-possibilities constraint:

$$a^i_1 S^i_1 + a^i_2 S^i_2 = Z^i, \text{ where } S^i_j \geq 0. \tag{2}$$

As in the standard model, comparative advantage is due to international differences in productivity and not to differences in factor endowments.[19] To fix ideas, we assume that country i has a comparative advantage in good i; that is, $a^i_i/a^i_j < a^j_i/a^j_j$ for $i \neq j = 1, 2$, which states that the opportunity cost in country i in producing good i is lower than the corresponding opportunity cost in country j $(\neq i)$. Furthermore, to economize on notation

[16] Our analysis, with a focus on just two consumption goods, considers adjustments only at the intensive margin, but could be extended to consider a continuum of consumption goods, as in Dornbusch et al. (1977). We chose the two-good version of the model in view of researchers' general familiarity with it and because it reveals the possibility that trade in the presence of conflict can be welfare reducing.

[17] Assuming CES preferences is not critical here, but helps simplify notation.

[18] More precisely, μ^i is the inverse of the price index that is dual to the CES aggregator.

[19] We impose this production structure with an intermediate input for convenience. An analytically equivalent approach would be to assume that the production functions of traded goods in each country differ by a Hicks neutral factor of proportionality, but not in factor intensities. Importantly, our approach isolates the TOT effects on arming by abstracting from factor-endowment based rationales for trade patterns and possible adjustments in arming decisions due to trade related factor-price effects (that have already been studied in extensions of the Heckscher-Ohlin model—e.g., Garfinkel et al., 2015).

and emphasize the importance of other variables of interest, we normalize $a_i^i = 1$ and let $\alpha^i \equiv a_j^i > 1$.

Trade regimes and product prices. Naturally, the allocation of Z^i across the two industries depends on product prices and thus on the trade regime in place. Let c^i denote the cost of producing one unit of Z^i. (We will describe the properties of c^i shortly.) Each country i's income Y^i is given by $Y^i = c^i Z^i$.

Under autarky, both consumption goods must be produced domestically. Then, competitive pricing in product markets requires $p_i^i = c^i$ and $p_j^i = \alpha^i c^i$, which give the equilibrium relative price of good j under autarky: $p_A^i = p_j^i/p_i^i = \alpha^i$, a constant. In turn, with the demand functions (1), p_A^i determines the division of income between goods. This division along with the product market-clearing requirement that $D_j^i = S_j^i$ for each good j determines (by (2)) equilibrium quantities.

Turning to the case of free trade, perfect competition requires once again that $p_i^i = c^i$, since country i always produces good i. Furthermore, absent trade costs, perfect competition requires $p_j^i = \min[\alpha^i c^i, c^j]$ for $i \neq j = 1, 2$.[20] It follows that at least one country will cease to produce the good in which it has a comparative disadvantage so that $p_j^i = p_j^j = c^j$ for $i \neq j = 1$ and/or 2. For now, let us suppose that each country i's demand for good j is fulfilled entirely by imports. Then, world prices that clear the market for good j satisfy $D_j^i + D_j^j = S_j^j = Z^j$ for $j\ (\neq i)\ = 1, 2$. Since under free trade consumers face identical prices and consumer preferences are identical across countries, their expenditure shares on each good will not differ: $\gamma_j^i = \gamma_j^j = \gamma_j$ for $j \neq i = 1, 2$. Maintaining our focus on the market for good j, now let $p_T^i \equiv p_j^i/p_i^i$ denote the relative domestic (and, since trade is free, world) price of country i's imported good j ($\neq i$). Substitution of the demand functions (1) with the competitive pricing relations into the market-clearing condition implies

$$p_T^i = \gamma_j Z^i/\gamma_i Z^j, \quad i \neq j = 1, 2, \tag{3}$$

where $\gamma_j = (p_T^i)^{1-\sigma}/[1+(p_T^i)^{1-\sigma}]$ and $\gamma_i = 1-\gamma_j$.[21] This expression reveals that the relative price of good j for country i under free trade depends on its supply of the intermediate good relative to that of its rival, Z^i/Z^j. More precisely, let a hat "ˆ" over a variable denote its percentage change (e.g., $\widehat{x} \equiv dx/x$); then, by logarithmically differentiating (3), while

[20] To keep the analysis as transparent as possible, we abstract for now from trade costs (tariffs and/or non-tariff trade barriers). We explicitly consider such costs in Section 4.2.

[21] Another way to obtain (3) is to set world relative demand for good j (i.e., $(D_j^i+D_j^j)/(D_i^i+D_i^j) = \gamma_j/\gamma_i p_T^i$) equal to its corresponding world relative supply (i.e., $S_j^j/S_i^i = Z^j/Z^i$) and then solve for p_T^i. Of course, depending on endowments, technology, and consumer preferences, p_T^i could be determined by one country's autarkic price. We revisit this possibility later.

accounting for the dependence of the expenditure shares on p_T^i, one can confirm

$$\widehat{p}_T^i = \frac{1}{\sigma}\left[\widehat{Z}^i - \widehat{Z}^j\right], \quad i \neq j = 1, 2. \tag{4}$$

Thus, an exogenous increase in Z^i/Z^j expands the relative supply of country i's exported good, thereby worsening its TOT, p_T^i.

2.2 Conflict and the Endogeneity of Intermediate Inputs

Having described the baseline model, we now introduce conflict between the two countries over contested capital K_0. After specifying the technologies for producing guns G^i and the intermediate input Z^i, we derive the equilibrium (Z^1, Z^2) for given (G^1, G^2)

Conflict technology. With the ultimate goal of maximizing national welfare V^i, each country i chooses its guns G^i to contest K_0 and thereby expand its capacity to produce consumption goods. More precisely, we assume that country i secures a share ϕ^i of K_0 that depends on guns produced by both countries as follows:

$$\phi^i = \phi(G^i, G^j) = \frac{f(G^i)}{f(G^i) + f(G^j)}, \quad i \neq j = 1, 2, \tag{5}$$

where $f(\cdot) > 0$, $f(0)$ is arbitrarily close to 0, $f'(\cdot) > 0$, and $f''(\cdot) \leq 0$. This specification of the conflict technology, also known as the "contest success function" (CSF), implies that country i's share of K_0 is increasing in its own guns ($\phi^i_{G^i} > 0$) and decreasing in the guns of its adversary ($\phi^i_{G^j} < 0$, $j \neq i$). Moreover, ϕ^i is symmetric; so $G^1 = G^2 \geq 0$ implies $\phi^1 = \phi^2 = \frac{1}{2}$.[22] The influence of guns on the division of K_0 between the two countries can be thought of as the result of either open conflict (without destruction) or a bargaining process with the countries' relative military strength playing a prominent role.[23]

Technologies for guns and intermediate goods. Guns are produced with *secure* labor and/or capital endowments. Letting w^i and r^i denote the competitive rewards paid to labor and capital in country i respectively, define $\psi(w^i, r^i)$ as the cost of producing one gun in country i. This unit cost function, which is identical across countries, is increasing, concave and homogeneous of degree one in factor prices. By Shephard's lemma, its partial derivatives ψ_w^i and ψ_r^i give the conditional input demands in the production of one gun. Thus, the quantities of resources diverted to contesting K_0 in each country i are $\psi_w^i G^i$ and

[22]See Skaperdas (1996), who axiomatizes a similar specification that is standard in the contest and conflict literatures. Our slight modification, requiring $f(0)$ to be positive but arbitrarily close to 0, is helpful in our proofs of existence of equilibrium in arming choices. (One example is $f(G) = (\delta + G)^b$ with $b \in (0, 1]$ and $\delta > 0$.) More generally, the appeal of this specification for our analysis derives from the fact that, because it is well understood, it allows us to highlight the effects of the option to trade for arming incentives.

[23]See Anbarci et al. (2002) who study how alternative bargaining solution concepts translate into rules of division that differ in their sensitivity to guns. Also see Garfinkel and Syropoulos (2018) for a related analysis in a trade-theoretic setting.

$\psi_r^i G^i$, with ψ_r^i/ψ_w^i representing the corresponding capital-labor ratio in the guns sector.[24]

Once guns have been produced and the disputed resource has been divided according to (5), the residual quantities of labor and capital available to country i to produce Z^i are respectively $L_Z^i = L_g^i - \psi_w^i G^i$ and $K_Z^i = K_g^i - \psi_r^i G^i$, where $L_g^i \equiv L^i$ and $K_g^i \equiv K^i + \phi^i K_0$ denote "gross" quantities of these primary resource factors. We can now identify the unit cost c^i of producing Z^i introduced above with $c(w^i, r^i)$.[25] This function, too, is identical across countries, increasing, concave and homogeneous of degree one in factor prices. Furthermore, $L_Z^i = c_w^i Z^i$ and $K_Z^i = c_r^i Z^i$ represent the relevant input demand functions, with c_r^i/c_w^i giving the capital-labor ratio demanded in the production of Z^i.

Determination of intermediate goods. Factor-market clearing in each country i $(= 1, 2)$ requires

$$c_r^i Z^i + \psi_r^i G^i = K_g^i \tag{6a}$$

$$c_w^i Z^i + \psi_w^i G^i = L_g^i. \tag{6b}$$

Our assumptions on the unit cost functions imply that the above system of equations can be solved to obtain the relative wage, ω^i $(\equiv w^i/r^i)$, and the quantity of Z^i available for the production of consumption goods. We describe this solution as follows. For given guns, we rearrange and combine (6a) and (6b) to obtain

$$\frac{c_r^i}{c_w^i} = \frac{K_g^i - \psi_r^i G^i}{L_g^i - \psi_w^i G^i}. \tag{7}$$

The right-hand side (RHS) of the above equation represents the capital/labor ratio $k_Z^i \equiv K_Z^i/L_Z^i$ supplied, whereas the left-hand side (LHS) represents the value of k_Z^i demanded. Condition (7) implicitly defines the equilibrium wage-rental ratio $\omega^i \equiv w^i/r^i$ in country i as a function of K_g^i, L_g^i, and G^i to ensure factor-market clearing—that is, $\omega^{ie} \equiv \omega^{ie}(K_g^i, L_g^i, G^i)$.

By our assumption of perfect competition, the value of country i's production of the intermediate input (denoted by R^i, which coincides with the value of domestic production) equals $c^i Z^i$. Equations (6a) and (6b), together with the linear homogeneity of the unit cost functions, imply $R^i = w^i L_g^i + r^i K_g^i - \psi^i G^i$. In turn, since $Z^i = R^i/c^i$, we have

$$Z^i(\omega^i, K_g^i, L_g^i, G^i) = \frac{\omega^i L_g^i + K_g^i - \psi(\omega^i, 1) G^i}{c(\omega^i, 1)}. \tag{8}$$

[24]Throughout, we assume that the secure labor and capital resource constraints do not bind in the production of guns for either country i: $L^i - \psi_w^i G^i > 0$ and $K^i - \psi_r^i G^i > 0$.

[25]Henceforth, we assume $c(w^i, r^i) \neq \psi(w^i, r^i)$, thereby embedding a neoclassical structure in an otherwise classical model, rich with political economy implications. More specifically, building on the model's mechanism (derived below) that connects factor rewards to arming, one could generate novel insights into the possible linkages between the distribution of national income, defense and external conflict.

Then, using ω^{ie} implied by (7) in the RHS of (8) delivers $Z^{ie} \equiv Z^{ie}(K_g^i, L_g^i, G^i)$, which we refer to as the optimized production of the intermediate input. Given G^i, K_g^i and L_g^i, this optimized quantity is independent of the trade regime in place.[26] Observe especially that Z^{ie} depends not only on G^i directly, but also on guns produced by both countries indirectly through K_g^i. Thus, as we will see below, this function plays a pivotal role in our analysis of equilibrium security policies. Henceforth, to avoid notational cluttering, we omit the "e" designation in the superscript when referring to the equilibrium values of Z^i and ω^i.

2.3 Equilibrium and Payoff Functions Given Guns

Bringing the essential elements of the model together, the sequence of events is as follows:

Stage 1. Each country i $(= 1, 2)$ simultaneously (and noncooperatively) chooses its guns G^i to maximize V^i, using its secure endowments (L^i units of labor and K^i units of capital).

Stage 2. Once the contested resource K_0 is divided according to ϕ^i induced by the arming choices made in stage 1, each country i uses its remaining inputs (L_Z^i units of labor and K_Z^i units of capital) to produce the intermediate good, Z^i.

Stage 3. Each country i uses its output of Z^i to produce S_i^i and S_j^i units of consumption goods i and j respectively, which are traded domestically and/or internationally depending on the trade regime in place.

Having presented, given G^i $(i = 1, 2)$, the conditions for the equilibrium determination of Z^i in stage 2 (Section 2.2) and for the equilibrium allocation of Z^i across sectors $j = 1, 2$ in each country i in stage 3 contingent on the trade regime in place (Section 2.1), we turn to study the non-cooperative, subgame perfect equilibria in security policies (i.e., the determination of guns) in stage 1. As the final step in preparing for this analysis, we now derive the trade-regime dependent payoff functions $V^i = \mu^i Y^i$ for each country $i = 1, 2$.

To that end, recall that, since country i always produces the good in which it has a comparative advantage, $p_i^i = c^i$ necessarily holds. That competitive pricing relationship implies, in turn, $\mu(p_i^i, p_j^i)c^i = \mu(1, p^i)$. For what follows, define $m(p^i) \equiv \mu(1, p^i)$, where $m'(p^i) < 0$. Then, recalling that $Y^i = c^i Z^i$, where Z^i satisfies (7) and (8), allows us to write the payoffs V_J^i under autarky $(J = A)$ and free trade $(J = T)$ as follows:

$$V_J^i = m(p_J^i) Z^i(K^i + \phi^i K_0, L^i, G^i), \quad \text{for} \quad i \neq j = 1, 2. \tag{9}$$

An important feature of country i's payoff function under autarky (V_A^i) is that, since $p_A^i = \alpha^i$ is constant, so is $m(p_A^i)$. Thus, each country i's arming decision under autarky that maximizes V_A^i depends solely on how G^i influences the optimized value of the intermediate input, Z^i. Guns production influences the payoffs under trade V_T^i, like those under autarky V_A^i,

[26] See Lemma A.1 (presented in Appendix A) that also shows how Z^{ie} and ω^{ie} depend on K_g^i, L_g^i, and G^i.

through its effect on Z^i. However, country i's production of guns influences V^i_T through an additional channel—namely, through its effect on the relative price of country i's imported good, p^i_T. Specifically, from (4), an increase in G^i, given G^j, expands Z^i and reduces Z^j, and thereby raising p^i_T. This deterioration of country i's TOT alone reduces its payoff under trade V^i_T through $m(p^i)$.

While one of our primary goals in this paper is to explore the trade-regime dependence of arming incentives and the associated welfare implications, it is instructive to see how payoffs under autarky and trade compare for given guns. Based on a standard, gains-from-trade argument using (9) for $J = A, T$, one can establish the following:

Lemma 1 *For any given feasible guns and gross factor endowments, payoffs under autarky and trade are ranked as follows:* $V^i_A \leq V^i_T$, *for each* $i = 1, 2$.

Intuitively, when country i's cost of importing good j (p^i_T) does not differ from its opportunity cost of producing the good itself ($\alpha^i = p^i_A$), country i produces both goods locally, implying it obtains identical payoffs under the two trade regimes. Trade flows between the two countries will be strictly positive, given guns, only when $p^i_T \leq \alpha^i$ (with strict inequality for at least one country) to make both countries at least as well off and at least one country strictly better off under trade than under autarky.[27] Any payoff increases due to a shift to free trade (given arming) reflect the *familiar gains from trade* that follow from canonical trade models based on comparative advantage.

3 Endogenous Security Policies

We now turn to the determination of non-cooperative, subgame perfect equilibria in security policies and their dependence on trade regimes. Inspection of the objective functions under autarky ($J = A$) and under trade ($J = T$) in (9) reveals that the equilibrium production of the intermediate input, represented by the envelope function Z^i in (8), is of central importance here. As noted earlier, given the countries' guns choices, Z^i is independent of the prevailing trade regime. Thus, the trade-regime dependence of arming incentives operates solely through a TOT channel.

To set the stage for this analysis, we make two additional observations that can be verified from (8).[28] First, the effect of a marginal increase in country i's gross endowment of capital $K^i_g = K^i + \phi^i K_0$ on Z^i is given by $Z^i_K = r^i/c^i$ and the effect of a marginal increase in its arming G^i (given K^i_g) on Z^i is $Z^i_{G^i} = -\psi^i/c^i$. Second, an increase in G^i also affects the rival's optimized production of the intermediate good Z^j through its influence on the rival's gross capital endowment, K^j_g: $Z^j_K = r^j/c^j$. Bringing these observations together,

[27] Of course, if $p^i_T < p^i_A = \alpha^i$ holds for both countries, then each country i specializes completely in the production of good i and both are strictly better off under trade.

[28] Also see the proof of Lemma A.1 presented in Supplementary Appendix B.1

while noting from (5) $\phi^j_{G^i} = -\phi^i_{G^i}$, shows:

$$\frac{dZ^i}{dG^i} = K_0\phi^i_{G^i} Z^i_K + Z^i_{G^i} = \frac{1}{c^i}\left[r^i K_0 \phi^i_{G^i} - \psi^i\right] \tag{10a}$$

$$\frac{dZ^j}{dG^i} = K_0\phi^j_{G^i} Z^j_K = -\frac{1}{c^j}[r^j K_0 \phi^i_{G^i}], \tag{10b}$$

given G^j, for $i \neq j = 1, 2$.

3.1 Autarky

As revealed by the payoff functions under autarky shown in (9) for $J = A$ where $m(p^i_A) = m(\alpha^i)$ is a constant, $\widehat{V}^i_A = \widehat{Z}^i$ holds. Thus, country i's arming choice influences V^i_A only through its effect on the maximized value of country i's intermediate input, Z^i. From (10a), each country i's first-order condition (FOC) for the choice of guns G^i, taking G^j as given, can be written as

$$\frac{1}{m(\alpha^i)}\frac{\partial V^i_A}{\partial G^i} = \frac{dZ^i}{dG^i} = \frac{1}{c^i}\left[r^i K_0 \phi^i_{G^i} - \psi^i\right] \leq 0, \quad i = 1, 2. \tag{11}$$

The first term in the brackets on the RHS of (11) reflects the marginal benefit of producing an additional gun for country i. Specifically, given G^j, an increase in G^i increases the share of the disputed resource K_0 that country i captures in the contest, thereby increasing its income and payoff. However, as shown in the second term, that additional gun reduces country i's income as it diverts resources away from the production of Z^i. Each country's optimal security policy balances this trade-off at the margin. Importantly, the negative influence of country i's security policy on country j's payoff through its effect on Z^j (shown in (10b)) does not directly enter this calculus.

Maintaining focus on the case in which the secure resource constraints on guns production do not bind, equation (11) is an exact statement of country i's FOC under autarky. Furthermore, the conflict technology (5) implies interior solutions with (11) holding as an equality. Since Z^i is concave in the country's own guns G^i and in the country's gross capital endowment K^i_g (see parts (c) and (e) of Lemma A.1 presented in Appendix A) and K^i_g is concave in G^i through the conflict technology (5), we can show that V^i_A is strictly quasi-concave in G^i. This property ensures the existence of an interior (pure-strategy) equilibrium in security policies. Let (G^1_A, G^2_A) be an equilibrium pair of guns. Some additional (but relatively mild) assumptions imply the equilibrium is unique:

Proposition 1 (*Equilibrium security policies under autarky.*) *An interior equilibrium in security policies exists under autarky: $G^i_A > 0$, for $i = 1, 2$. Furthermore, if labor and capital are sufficiently substitutable in the production of arms and/or the intermediate good, this equilibrium is unique.*

Since each country's problem under autarky is effectively one of maximizing income or, equivalently, the quantity of the intermediate good used in the production of traded goods, the equilibrium in security policies under autarky is independent of the elasticity of substitution in consumption. Matters differ, however, in the case of trade.

3.2 Free Trade

As previously discussed, security policies under trade, like those under autarky, affect payoffs through their impact on the output levels of the intermediate good. However, when trade is possible, these output changes also affect world prices as shown in (4). Using (4) and (9) for $J = T$ and noting that $\frac{m'(p_T^i)}{m(p_T^i)} p_T^i = -\gamma_j$, one can verify the following:

$$\widehat{V}_T^i = \widehat{Z}^i + \left[\frac{m'(p_T^i)}{m(p_T^i)} p_T^i\right] \widehat{p}_T^i = \widehat{Z}^i - \frac{\gamma_j}{\sigma}\left(\widehat{Z}^i - \widehat{Z}^j\right), \quad i \neq j = 1, 2. \tag{12}$$

The second term in the most RHS of (12) captures the TOT effect of changes in (Z^i, Z^j).

Combining (10) with (12), the FOC for country i's arming choice becomes:

$$\frac{1}{m(p_T^i)} \frac{\partial V_T^i}{\partial G^i} = \frac{1}{c^i}\left\{\left[1 - \frac{\gamma_j}{\sigma} - \left(\frac{r^j/c^j Z^j}{r^i/c^i Z^i}\right)\frac{\gamma_j}{\sigma}\right] r^i K_0 \phi_{G^i}^i - \left[1 - \frac{\gamma_j}{\sigma}\right]\psi^i\right\} \leq 0, \tag{13}$$

for $i = 1, 2$ and $j \neq i$. Similar to the FOC under autarky (11), this FOC consists of two components: the marginal benefit and the marginal cost of an additional gun. However, the negative effect of an additional gun on country i's TOT modifies these two components substantively as compared with autarky, due to the negative effect of an increase in i's guns on the rival's output Z^j.

To characterize the equilibrium under trade, we henceforth assume that $\sigma > \gamma_j$ for $j = 1, 2$, so that the possibility of immiserizing growth (Bhagwati, 1959) is ruled out.[29] This assumption, however, does not ensure that the marginal benefit of arming, when evaluated at $G^i = 0$, is strictly positive. Digging a little deeper, let us define

$$\zeta^j \equiv \frac{r^i/c^i Z^i}{r^j/c^j Z^j + r^i/c^i Z^i} = \frac{c^j Z^j/r^j}{c^i Z^i/r^i + c^j Z^j/r^j}, \text{ for } i \neq j.$$

The function ζ^j reflects country j's relative size in terms of the countries' GDP, net of arming and measured in domestic units of the insecure resource. Then, the sign the marginal benefit for country i (i.e., the first term in (13)) is determined by the sign of $\sigma - \gamma_j/\zeta^j$. Accordingly, there exists a critical value of σ, $\overline{\sigma}^i \equiv \gamma_j/\zeta^j$ evaluated at $G^i = G^j = 0$ for each country $i \neq j = 1, 2$, such that country i's marginal benefit of arming when $G^i = 0$ is strictly

[29] The possibility of immiserizing growth requires $dV_T^i/dZ^i < 0$ and normally arises if the elasticity of substitution in consumption is sufficiently low. However, from (12), we have $\text{sign}\{dV_T^i/dZ^i\} = \text{sign}\{1-\gamma_j/\sigma\}$, which is positive for all equilibria (interior or not) provided $\sigma > \gamma_j$.

positive (non-positive) if $\sigma > \overline{\sigma}^i$ ($\sigma \leq \overline{\sigma}^i$).

Since $\gamma_1 + \gamma_2 = \zeta^1 + \zeta^2 = 1$, there are two distinct possibilities to consider: (i) $\overline{\sigma}^1 = \overline{\sigma}^2 = 1$, which arises when the two countries have identical secure endowments;[30] and, (ii) $\overline{\sigma}^i > 1 > \overline{\sigma}^j$ for $i \neq j = 1, 2$, which arises in the presence of asymmetries. Thus, if $\sigma \leq 1$, the marginal benefit of arming must be non-positive for at least one country i (the relatively larger one) and possibly both; if $\sigma > 1$, the marginal benefit of arming must be positive for at least one country j (the relatively smaller one) and possibly both. Also observe that the maximum value of $\overline{\sigma}^i$ across i, $\overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$, is greater than or equal to 1.

Building on these ideas and using the FOC under trade (13), it is possible to verify that an equilibrium always exists. However, multiple equilibria in pure and mixed strategies are possible. Nonetheless, provided the strength of the two countries' comparative advantage, represented by α^i for $i = 1, 2$, are sufficiently high, a pure-strategy equilibrium that differs from the one under autarky exists under free trade.[31] Letting (G_T^1, G_T^2) denote that equilibrium, we have

Proposition 2 (*Equilibrium security policies under free trade.*) *Suppose the conditions that ensure a unique equilibrium under autarky are satisfied and each country's comparative advantage (α^i) is sufficiently strong. Then, there exists a pure-strategy equilibrium in security policies under free trade that is distinct from the equilibrium under autarky:* (*i*) $G_T^i = 0$ *for* $i = 1, 2$ *if* $\sigma \leq \overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$ *and* (*ii*) G_T^i ($\neq G_A^i$) > 0 *for* $i = 1, 2$ *if* $\sigma \in (\overline{\sigma}, \infty)$.

Observe that the elasticity of substitution in consumption (σ) plays an essential role under free trade, as it determines the magnitude of the TOT effect. If the two goods are distant substitutes (i.e., $\sigma \leq \overline{\sigma}^i$ holds for one country $i = 1, 2$ or both), then country i's payoff under trade is highly dependent on country j's production, through relative world prices. Even though an increase in guns by country i, where initially $G^i = 0$ and given $G^j \geq 0$, generally implies a positive net marginal benefit of arming for given world prices (i.e., $r^i K_0 \phi^i_{G^i} - \psi^i > 0$), that additional gun implies at the same time a worsening of its TOT. More precisely, it brings about both an increase in the supply of its exported good and a decrease in the supply of its imported good. This negative TOT effect tends to reduce the effective marginal benefit of an additional gun by more than it reduces the marginal

[30] Specifically, in a symmetric equilibrium, we have $Z^i = Z^j$ which implies $p_T^i = 1$ and thus $\gamma_i = \gamma_j = \frac{1}{2}$ for $i = 1, 2$ (see the proof of Proposition 3 for some details). Since equilibrium factor prices are also identical across countries, $\zeta^j = \frac{1}{2}$ holds in this benchmark case. Thus, $\sigma > 1$ ($\sigma \leq 1$) implies that the marginal benefit of arming is strictly positive (non-positive) at $G^i = 0$ for both i.

[31] The potential problem, as analyzed in Supplementary Appendix B.1, is that p_T^i can vary only within the range $[1/\alpha^j, \alpha^i]$, giving rise to a possible discontinuity in country i's best-response function under trade such that it coincides with country i's best-response function under autarky for some G^j. Requiring that comparative advantage α^i be sufficiently strong for both $i = 1, 2$ ensures the existence of a pure-strategy equilibrium in security policies that is distinct from the equilibrium under autarky. But, even when this requirement is satisfied, we cannot rule out the possible existence of multiple equilibria in pure strategies.

cost. When $\sigma < \overline{\sigma}^i$, our specification of the conflict technology (5), with $\phi^i_{G^i} \in (0,\infty)$ at $G^i = 0$, implies country i chooses to produce no guns at all. If, in addition, $\sigma \leq \overline{\sigma}^j$, then $G^j_T = G^i_T = 0$. But, even if $\sigma > \overline{\sigma}^j$ while $\sigma \leq \overline{\sigma}^i$, our specification of the conflict technology (5) requiring that $f(0)$ be arbitrarily close to zero (even if positive) implies that country j's best response to $G^i = 0$ is to produce an infinitesimal amount of guns. Therefore, when $\sigma \leq \overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$, we have $G^1_T = G^2_T = 0$.[32]

When the two consumption goods are sufficiently substitutable (i.e., $\sigma > \overline{\sigma} \geq 1$), the negative effect through the TOT channel is not large enough to wipe out the positive net marginal benefit of arming given world prices when evaluated at $G^i = 0$ for either country i. The conditions specified in the proposition along with (5) ensure, in this case, the existence of an interior equilibrium in security policies under trade that differs from the equilibrium under autarky. What's more, the externality of each country j's arming on the rival's payoff remains negative in this equilibrium: $dV^i_T/dG^j < 0$.[33]

4 Equilibrium Arming and the Relative Appeal of Trade

We are now in a position to compare the equilibrium payoffs under the two trade regimes, denoted by $V^{i*}_A \equiv V^i_A(G^i_A, G^j_A)$ in the case of autarky and $V^{i*}_T \equiv V^i_T(G^i_T, G^j_T)$ in the case of trade, for $i \neq j = 1, 2$. Combining Lemma 1 with the equilibrium analysis underlying Propositions 1 and 2, the next lemma establishes that a sufficient condition for trade to dominate autarky is simply that trade results in lower arming by both countries:

Lemma 2 (*Equilibrium payoffs under autarky vs. free trade.*) *Suppose that trade induces both countries to arm by less as compared with autarky. Then, each country is strictly better off under free trade than under autarky (i.e., $V^{i*}_T > V^{i*}_A$ for $i = 1, 2$), and the difference in payoffs for both exceeds each country's standard gains from trade.*

The potential benefit of moving to free trade can be decomposed into three parts. First, given both countries' arming choices, each country can enjoy the standard gains from trade that are, by Lemma 1, non-negative for both countries and strictly positive for at least one. Second, each country enjoys, given its own arming choice, a positive strategic effect as the opponent reduces its arming. Finally, each country's payoff rises, as it optimally adjusts

[32] Observe that $\sigma \leq 1$ is sufficient (but not necessary) for trade to effectively remove both countries' incentive to arm. Furthermore, while our assumption that $\sigma > \gamma_j$ rules out immizerising growth as a possible explanation for reduced arming incentives under trade in this analysis, lower values of σ do imply more generally both a greater likelihood of such a phenomenon and lower incentives to arm.

[33] One can confirm this claim, which also holds true when $\overline{\sigma}^i > \sigma > 1$, by evaluating

$$\frac{\partial V^i_T/\partial G^j}{V^i} = \frac{dZ^i/dG^j}{Z^i} - \frac{\gamma_j}{\sigma}\left[\frac{dZ^i/dG^j}{Z^i} - \frac{dZ^j/dG^j}{Z^j}\right],$$

at the value of $\frac{dZ^i/dG^j}{Z^i}$ from (12) implied by country j's FOC under trade for an interior solution, to find that a necessary and sufficient condition for $\partial V^i_T/\partial G^j < 0$ is that $\sigma > 1$.

its own arming choice in response to the new trade regime. Of course, in the presence of asymmetries in secure resources, one country could become less powerful under trade relative to its position under autarky. However, provided that both countries reduce their arming, each realizes lower security costs on top of the standard gains from trade.

4.1 Arming under Autarky vs. Free Trade

Using Propositions 1 and 2, we now study the difference in equilibrium guns chosen by the two adversaries under the two trade regimes. Proposition 1 established that, under autarky, $G_A^i > 0$ for $i = 1, 2$. By contrast, Proposition 2 showed that, under free trade, $G_T^i = 0$ for $i = 1, 2$ when $\sigma \leq \overline{\sigma}$.[34] Thus, when the two goods are sufficiently distant substitutes in consumption, equilibrium arming is lower under free trade and, by Lemma 2, each country's welfare is higher; countries enjoy not only the standard gains from trade, but also the elimination of security costs it induces.

When the two consumption goods are sufficiently substitutable (i.e., $\sigma > \overline{\sigma}$), however, each country's optimizing choice of guns is strictly positive under both autarky and free trade. But, a comparison of the FOCs under autarky (11) and trade (13) reveals the adverse TOT effect of a country's own arming reduces the marginal benefit of an additional gun relative to the analogous marginal benefit under autarky by more than it reduces the relative marginal cost. Thus, for any given G^j, country i's best response under free trade is strictly less than its best response under autarky: $B_T^i(G^j) < B_A^i(G^j)$ for any $G^j \geq 0$, $i \neq j = 1, 2$. This finding gives us a sufficient but not necessary condition for trade to induce lower equilibrium arming when arming is strictly positive under both trade regimes. Specifically, if neither country's security policy exhibits strategic substitutability in the neighborhood of the autarkic equilibrium, then there exists a pure-strategy equilibrium under free trade where both countries choose lower arms.

Whether a country's best-response function is positively or negatively sloped in the neighborhood of the autarkic equilibrium depends on how an increase in the opponent's guns G^j influences country i's marginal benefit and marginal cost of arming.[35] An increase in G^j (given G^i) reduces country i's gross capital (K_g^i) and, thus, reduces its relative wage (ω^i) and so its marginal cost of arming. This effect alone induces country i to produce more guns as G^j rises. The effect of an increase in G^j on country i's marginal benefit of arming, however, can be positive or negative. Specifically, the conflict technology (5) implies $\phi_{G^iG^j}^i \gtreqless 0$ as $B_A^i(G^j) \gtreqless G^j$. Thus, when $B_A^i(G^j) > G^j$ ($B_A^i(G^j) < G^j$), the implied positive (negative) effect of G^j on country i's marginal benefit of arming alone induces it to produce more (less) guns.

This discussion suggests that the sufficient condition for trade to lower equilibrium

[34]As noted above, arming by one country could be strictly positive, but would be infinitesimal.

[35]See equation (A.5) in Appendix A that unveils the determination of the sign of the slope of $B_A^i(G^j)$.

arming and enhance each country's payoff can be traced back to fundamentals that influence relative arming by the two countries under autarky—namely, the distribution of secure resources.[36] Building on this idea with the assumption that the conditions of Propositions 1 and 2 are satisfied, we establish the following:

Proposition 3 (*Secure resource distributions and a comparison of arming.*) *Suppose the elasticity of substitution in consumption is sufficient large (i.e., $\sigma > \overline{\sigma}$). Then, there exists a set of secure resource distributions with a sufficiently even mix of labor and capital across countries i such that $G_A^i > G_T^i > 0$ for $i = 1, 2$.*

The set of resource distributions for which trade induces lower equilibrium arming includes the benchmark case that we refer to as *complete symmetry*: countries have identical initial (secure) resource endowments in addition to identical preferences (defined symmetrically over the two consumption goods) and technologies for producing Z^i and G^i. Since they arm identically under autarky, $\phi^i_{G^iG^j} = 0$ holds, and both countries' security policies necessarily exhibit strategic complementarity in the neighborhood of the autarkic equilibrium, as illustrated in Fig. 1(a). Starting at point A (i.e., where $G_A^i = G_A > 0$ for $i = 1, 2$), a shift to free trade induces both countries' best-response functions to rotate at the origin towards the 45° line and intersect at a new equilibrium with less arming by both countries (point T, where $0 < G_T^i = G_T < G_A$ for $i = 1, 2$).[37] As shown in the proof to this proposition, there also exist asymmetric distributions of secure resources that similarly imply a symmetric equilibrium in security policies under autarky and thus satisfy the sufficient condition for trade to induce lower arming. Furthermore, by continuity, there exist other asymmetric distributions adjacent to that set, which imply asymmetric equilibria under autarky and hence $\phi^i_{G^iG^j} < 0$ for one country i, but for which the positive effect of an increase in G^j on country j's marginal cost of arming dominates. Hence, both countries' security policies continue to exhibit strategic complementarity, such that once again a shift from autarky to trade implies lower arming by both countries. It follows from Lemma 2 that, for distributions associated with a sufficiently even mix of secure resources, a shift to free trade is welfare-improving for both countries; what's more, their gains from trade are strictly greater than those predicted by the traditional paradigm that abstracts from conflict altogether.

Of course, we cannot rule out the possibility that a shift from autarky to free trade implies greater arming by one country. For such an outcome to arise, the mix of secure

[36] The fact that relative endowments are observable and contain information regarding differences in the degree of resource security across countries makes this focus appealing over considering, for example, differences in the countries' technologies for producing guns or asymmetries in the conflict technology.

[37] Proposition B.1 presented in Supplementary Appendix B.1 shows G_T is increasing in the elasticity of substitution in consumption for $\sigma > 1$ $(= \overline{\sigma})$, approaching G_A (which is independent of σ) as $\sigma \to \infty$. Thus, consistent with Hirshleifer's (1991) argument, the savings in security costs afforded by trade equal zero when the two goods are perfect substitutes and increase as σ falls approaching 1 (or equivalently as the two economies become increasingly "integrated").

labor and capital resources held initially by the two countries must be sufficiently uneven to generate large differences in their arming choices under autarky that imply $\partial B^i_A/\partial G^j < 0$ for $i \neq j = 1$ or 2.[38] But, even in this case, we could have $G^i_T < G^i_A$ for $i = 1, 2$. A more extreme asymmetry in secure resource endowments across countries is required for trade to induce one country to become more aggressive, as illustrated in Fig. 1(b), which assumes that country 2 has the higher capital/labor ratio and thus arms less under autarky than its rival.[39] Starting from the autarkic equilibrium depicted by point A in the figure, a shift to free trade causes each country's best-response function to rotate at the origin inward towards the 45^o line, resulting in a new intersection at point T, with decreased arming by country 1 but increased arming by country 2.

What are the welfare implications of trade when trade induces one country (2) to expand its arming? Because country 1 arms less heavily under trade than under autarky, country 2 enjoys a positive strategic welfare effect as well as the standard gains with a move from autarky to trade. By contrast, due to the adverse strategic effect of country 2's arming, it is possible for trade to reduce country 1's payoff. Such a preference ranking is more likely to hold when country 1's strength of comparative advantage (α^1) and thus the standard gains it realizes from trade are relatively small. Either way, the possibility that trade could induce greater arming by one country under some circumstances illustrates one potential (although perhaps remote) limit to the classical liberal view.

4.2 The Effects of Trade Costs

Given our focus above on free trade, one might naturally wonder how trade costs matter. In this subsection we extend the analysis to study the importance of import tariffs and non-tariff trade barriers for arming and payoffs. Henceforth, to avoid complications related to discontinuities in best-response functions, we assume that, for any given secure resource endowments, the strength of each country i's comparative advantage (α^i) is sufficiently large and the initial levels of trade costs sufficiently low, so that each country i's internal prices differ from its autarky prices and thus its demand for good j is entirely satisfied by its imports: $M^i = D^i_j$ for $j \neq i = 1, 2$. Let τ^i and t^i respectively denote one plus an iceberg type trade cost and ad valorem tariff rate on its imports, and define p^i_T and q^i_T as the corresponding internal and external relative prices of the same product. Arbitrage implies that, in a trade equilibrium with positive trade flows, $p^i_T = \tau^i t^i q^i_T$ holds. As shown

[38] Equation (A.5) in Appendix A and the conflict technology (5) together imply that, at most, one country's best-response function can be negatively sloped in the neighborhood of the autarkic equilibrium.

[39] Note the scale for G^1 in the figure is more concentrated such that the slope of the "45^o" line drawn is greater than 45^o. Numerical simulations based on a particular parameterization of the model confirm that an increase in arming by one country is possible only under extremely uneven secure distributions of the two primary resources. Details are available from the authors upon request.

in Supplementary Appendix B.2, the percentage change in i's payoff V_T^i under trade is

$$\begin{aligned}\widehat{V}_T^i &= \left(1-\rho^i\right)\widehat{Z}^i + \rho^i \widehat{Z}^j - \rho^i\left[\varepsilon^j \widehat{\tau}^i + \left(\varepsilon^j - 1\right)\widehat{\tau}^j + \eta^j \widehat{t}^j\right] \\ &\quad + \left(\gamma_j^i / t^i \Delta\right)\eta^i\left[1 - \left(t^i - 1\right)\left(\varepsilon^j - 1\right)\right]\widehat{t}^i, \end{aligned} \tag{14a}$$

for $j \neq i = 1, 2$, where

$$\varepsilon^i \equiv -\frac{\partial M^i / \partial p_T^i}{M^i / p_T^i} = 1 + (\sigma - 1)\frac{t^i \gamma_i^i}{t^i \gamma_i^i + \gamma_j^i} \tag{14b}$$

$$\eta^i \equiv -\left.\frac{\partial M^i / \partial p_T^i}{M^i / p_T^i}\right|_{dU^i = 0} = \varepsilon^i - \frac{\gamma_j^i}{t^i \gamma_i^i + \gamma_j^i} = \sigma\left(\frac{t^i \gamma_i^i}{t^i \gamma_i^i + \gamma_j^i}\right) > 0 \tag{14c}$$

$$\Delta \equiv \varepsilon^i + \varepsilon^j - 1 \tag{14d}$$

$$\rho^i \equiv \left[\frac{\left(t^i - 1\right)\varepsilon^i + 1}{t^i}\right]\left(\frac{\gamma_j^i}{\Delta}\right). \tag{14e}$$

Note that ε^i (resp., η^i) is the absolute value of country i's Marshallian (compensated) price elasticity of import demand. Also note that $\text{sign}\{\varepsilon^i - 1\} = \text{sign}\{\sigma - 1\}$. For specificity and clarity, hereafter we focus on the case where $\sigma > 1$, which implies $\Delta > 1$ and $\rho^i \in (0, 1)$.

With (14) and our analysis of security policies under autarky we now arrive at

Lemma 3 (*Arming incentives under trade.*) *For any given guns $G^j > 0$, we have $B_T^i(G^j) < B_A^i(G^j)$ for $j \neq i = 1, 2$.*

Generalizing our analysis in Section 4.1, this lemma shows that a move from autarky to trade, whether free or distorted with trade costs, reduces each country's incentive to arm given the rival's guns. Strikingly, this result remains valid even if tariffs are chosen non-cooperatively and simultaneously with security policies.[40] Building on this lemma, one can establish the following:

Proposition 4 (*Equilibrium arming.*) *Aggregate equilibrium arming under trade is strictly less than that under autarky: $G_T^1 + G_T^2 < G_A^1 + G_A^2$. However, depending on consumer preferences, technology and the distribution of secure endowments, it is possible to have $G_T^i > G_A^i$ for one country $i \in \{1, 2\}$.*

Thus, as in the case of free trade, trade distorted with tariff and non-tariff barriers induces one adversary (and possibly both) to reduce its arming below the autarkic level. Still, it is possible for one adversary to produce more guns under trade than under autarky.[41]

[40] As in standard analyses of trade wars that abstract from resource disputes and the associated resource costs, autarky is always a possible equilibrium when tariff policies are chosen non-cooperatively; however, as suggested by the previous literature, an interior equilibrium in tariff policies normally exists even when countries differ in size (e.g., Johnson, 1953-54; Kennan and Riezman, 1988; and, Syropoulos, 2002).

[41] Numerical analysis confirms the possibility that one country arms by more under trade, when the international distribution of secure resources is extremely uneven (see Supplementary Appendix B.2).

Nevertheless, regardless of how it affects each country's security policy, trade always brings about a reduction in aggregate arming.

To get some sense of how trade costs influence arming incentives, let us return to our assumption of complete symmetry where the two countries have identical initial endowments of labor and capital, as well as technologies, but allow trade costs to differ across countries. Focusing on non-tariff (or non-revenue generating) trade costs, τ^i, we can establish

Proposition 5 (*Differences in non-tariff trade costs and relative arming under complete symmetry.*) *In a unique trade equilibrium, differences in non-tariff trade costs generate differences in equilibrium arming as follows:* $\tau^i \lesseqqgtr \tau^j$ *implies* $G_T^i \lesseqqgtr G_T^j$.

While this proposition considers a very special case, it allows us to identify a fundamental force at work here that can drive a wedge between equilibrium arming of two adversaries that are otherwise identical. Starting at the benchmark case where $\tau^i = \tau^j > 1$, the two countries arm identically under trade and, according to the logic presented above, by less than they would arm under autarky. A decrease in country i's trade costs τ^i leads to a reduction in each country's domestic price of the imported good (p_T^i and $p_T^j \downarrow$), thereby augmenting each country's dependence on imports from its rival (γ_j^i and $\gamma_i^j \uparrow$). However, these effects are more pronounced for country i, such that $p_T^i/p_T^j < 1$ and $\gamma_j^i/\gamma_i^j > 1$. As a result, $G_T^i < G_T^j$ for $\tau^i < \tau^j$. Furthermore, aggregate arming is less than when $\tau^i = \tau^j$, suggesting that a decrease in trade costs for one country amplifies trade's pacifying effects.

Moving beyond the case of identical adversaries, numerical analysis shows further that, under most circumstances, globalization (i.e., $\tau^i \downarrow$ for $i = 1$, 2 or both) reduces both countries' equilibrium arming.[42] To be sure, consistent with our analysis in Section 4.1, there do exist sufficiently uneven international distributions of secure resources to imply that the smaller country j's arming rises as τ^i falls (i.e., $dG_T^j/d\tau^i < 0$). However, exhaustive numerical analysis (assuming $\alpha^i = \infty$) confirms that, under all circumstances, aggregate arming falls and each country's payoff rises.

The analysis of tariffs (t^i) is, as one would imagine, more complex and thus more difficult to characterize in a succinct way. Specifically, the effect of an increase in t^i on arming depends not only on the distribution of secure resources but also on t^j and the initial level of t^i, giving rise to non-monotonicities in both equilibrium arming and payoffs. Nonetheless, based on numerical methods, we obtain some important results when comparing various equilibrium outcomes, including autarky (A), free trade (F) and "generalized war" (W), where (in the last case) both countries simultaneously and non-cooperatively choose their security and trade policies.[43] In particular, consistent with our priors based on the analysis

[42]Details are provided in Supplementary Appendix B.2.
[43]Again, see Supplementary Appendix B.2.

above and with the idea that trade wars have a prisoner-dilemma feature, we find that, under most circumstances, $G^i_A > G^i_W > G^i_F$ and $V^i_A < V^i_W < V^i_F$. However, when one country (i) is extremely larger than its rival (j), a different ranking of arming emerges for both countries. The smaller country (j) arms more heavily under free trade than under autarky, and it arms by even more under generalized war: $G^j_W > G^j_F > G^j_A$. Nonetheless, the ranking of its payoffs is the same as when the countries are more equally sized. While the much larger country (i) continues to arm by less under free trade than under autarky, its arming under generalized war is the lowest ($G^i_A > G^i_F > G^i_W$), suggesting that its extremely larger size renders its trade policy more effective in influencing its TOT. Furthermore, similar to Syropoulos' (2002) finding in a setting with secure property rights, we also find that this country prefers generalized war over the other outcomes: $V^i_A < V^i_F < V^i_W$. What is perhaps surprising is that this extremely large country prefers all these outcomes to those that would obtain if there were no resource insecurity and thus no arming at all.

5 Trading with Friends

While the analysis above provides analytical support to the notion that trade can be pacifying in international relations, our focus has been on two economically interdependent countries. In this section, we explore how the nature of the trading relationship between the contending nations can matter, showing a possible limit to the optimism of the classical liberal view. In particular, in contrast to our setting above where differences in technology to produce consumption goods render trade between adversaries mutually advantageous, we now consider a setting where the structure of technology is such that they do not trade with each other, but instead they trade with a third, non-adversarial country. Extending the analysis to a three-country, two-good model, we show that a shift to free trade can induce greater arming, even in the case of complete symmetry.[44]

Suppose consumers in all three countries ($i = 1, 2, 3$) have identical CES preferences defined over two consumption goods $j = 1, 2$, with $\sigma > 1$.[45] As before, the two adversarial countries ($i = 1, 2$) have identical technologies for producing guns to contest K_0 and the intermediate good; in addition, they have identical technologies for producing the two consumption goods: $a^1_1 = a^2_1 = 1$ and $a^1_2 = a^2_2 = a_2 > 1$, implying the relative price of good 2 under autarky in both countries satisfies $p^i_A = p_A = a_2$ for $i = 1, 2$. The third country ($i = 3$) is not involved in the contest over K_0 and, hence, considered "friendly." Its technology for producing the intermediate good is the same as that for the two adversaries; but its technology for producing consumption goods $j = 1, 2$ is described by $a^3_1 > a^3_2 = 1$, which implies $p^3_A = a^3_1$. Thus, $1/p^3_A < p_A$. Not surprisingly, the third country's presence

[44] We briefly consider the case of (possibly asymmetric) trade costs below.

[45] Although our results extend to the case where $\sigma = 1$ (with exceptions noted below), we maintain the assumption that $\sigma > 1$ for consistency with our analysis in Section 4.2 and for ease of exposition.

has no effect on arming incentives under autarky. Therefore, the resulting outcome is as characterized in Proposition 1.

When trade is possible, preferences and the production structure specified above imply countries 1 and 2 export good 1 in return for imports of good 2 from country 3. Under free trade, all countries face the same relative price for good 2, denoted by $p_T \equiv p_2/p_1$, and this price balances world trade. Since the countries have identical and homothetic preferences and face identical world prices, their expenditure shares are identical. Complete specialization in production implies that $Y^i = p_1 Z^i$ for $i = 1, 2$, and $Y^3 = p_2 Z^3$, such that $D_2^i = \gamma_2 Z^i / p_T$ $(i = 1, 2)$ and $D_1^3 = \gamma_1 p_T Z^3$. In turn, it implies that the world market-clearing condition is given by $p_T(D_2^1 + D_2^2) = D_1^3$. As such, $p_T = \gamma_2(Z^1 + Z^2)/\gamma_1 Z^3$. Now differentiate this expression logarithmically, keeping the intermediate output of country 3 (Z^3) fixed in the background, and rearrange to find: $\widehat{p}_T = \frac{1}{\sigma}(\nu^1 \widehat{Z}^1 + \nu^2 \widehat{Z}^2)$, where $\nu^i = Z^i/(Z^1 + Z^2)$ for $i = 1, 2$ represents country i's import share.

As in the two-country case, country i's security policy under trade affects the relative price of its imported good through its impact on Z^i for $i = 1, 2$, that satisfies (7) and (8). This effect, in turn, makes the incentive to arm trade-regime dependent.[46] Furthermore, as before, an increase in arming by country i reduces rival j's production of the intermediate good given G^j—i.e., $dZ^j/dG^i < 0$ for $i \neq j = 1, 2$, by (10b). The key difference here is that the decrease in Z^j now improves country i's terms of trade.

To see how this difference matters in determining arming incentives, note first that each adversary's payoff function under free trade V_T^i can be written as before and shown in (9) for $J = T$, but now $m(p_T^i) = m(p_T)$ with $\frac{m'(p_T)}{m(p_T)} p_T = -\gamma_2$ for $i = 1, 2$. Then, differentiate V_T^i to obtain

$$\widehat{V}_T^i = \widehat{Z}^i + \left[\frac{m'(p_T)}{m(p_T)} p_T\right] \widehat{p}_T = \widehat{Z}^i - \frac{\gamma_2}{\sigma}\left(\nu^i \widehat{Z}^i + \nu^j \widehat{Z}^j\right). \tag{15}$$

Using (10) in (15), country i's FOC under free trade can be written as:

$$\frac{1}{m(p_T)} \frac{\partial V_T^i}{\partial G^i} = \frac{1}{c^i}\left[\left(1 - \frac{\nu^i \gamma_2}{\sigma} + \frac{\nu^j \gamma_2}{\sigma}\left[\frac{r^j/c^j Z^j}{r^i/c^i Z^i}\right]\right) r^i K_0 \phi_{G^i}^i - \left(1 - \frac{\nu^i \gamma_2}{\sigma}\right)\psi^i\right] \leq 0, \tag{16}$$

for $i \neq j = 1, 2$. The first term inside the outer square brackets reflects country i's marginal benefit of arming, whereas the second term reflects its marginal cost. Assuming $\sigma > 1$ is sufficient to ensure that both terms are positive for each adversary. Furthermore, we assume sufficiently strong comparative that, in the case of symmetry where the two adversaries are

[46] Note, if the two adversarial countries were so small that their production of the intermediate input had no influence on p_T, their security policies would not affect prices, and therefore would not be trade-regime dependent. The sharp contrast of this result with that of Garfinkel et al. (2015) stems from the presence of a factor-price channel that is (intentionally) absent in the present analysis.

identical in terms of their initial secure resource endowments, ensures the existence of a unique equilibrium under free trade.[47] Then, using (16) and the FOC under autarky (11) for each country i, one can show the following:

Proposition 6 (*Equilibrium arming and payoffs under free trade with a third, friendly country.*) *Suppose that two identical adversarial countries potentially compete in the same market for exports to a third, friendly country. Then, a shift from autarky to free trade* (a) *induces each adversary to arm more heavily, and* (b) *can be welfare reducing for both.*

The intuition for part (a) is that, with trade, each country has an interest in producing more guns at the margin not only to appropriate more K_0 and thus produce more Z, but also to reduce its rival's output, thereby improving its TOT with its friendly trading partner.[48] Additionally, as one can verify, an increase in the elasticity of substitution in consumption (σ) reduces the magnitude of this TOT effect.[49] But, part (b) establishes, with intensified conflict between the two contending countries, trade brings higher security costs, and these higher security costs can swamp the gains from trade. In the proof, we establish this possibility based on comparative advantage that is just strong enough to ensure a unique equilibrium in arming under trade, while making the gains from trade very small.[50]

Applying our earlier logic from the two-country case shows both adversarial countries ($i = 1, 2$) produce more guns under costly trade than under autarky. We now ask how differences in non-tariff trade costs influence their arming choices, maintaining our focus on the case of identical adversaries.[51] Let us define $\tau^i \geq 1$ for $i = 1, 2$ as an iceberg type trade cost for i's imports from the friendly country and $\tau_i^3 \geq 1$ analogously for country 3's imports from country $i = 1, 2$. Differences in τ^i naturally lead to differences in import shares ν_2^i and expenditure shares γ_2^i. However, if we assume $\tau_i^3 = \tau_j^3$ ($i \neq j = 1, 2$) for simplicity, only the effects on γ_2^i matter for this comparison, allowing us to establish

[47] Assuming sufficiently strong comparative advantage once again allows us to focus on the case where the boundary conditions on the world price of country 1 and 2's imported good in terms of their exported good, $p_T \in (1/p_A^3, p_A)$ for $i = 1, 2$, do not bind, whereby we can abstract from the potential complications associated with discontinuities in the best-response functions as before. We do, however, discuss these issues in the detailed proof of Proposition 6(b), presented in Supplementary Appendix B.1.

[48] In its quest for raw resources with an aim to match Great Britain's access and thus be better able to compete with Great Britain in the export of manufactures to third-countries, Germany invaded parts of Eastern Europe at the outset of WWII. Eventually shifting its efforts westward, Germany had hoped that it could negotiate some sort of peaceful settlement with Great Britain. But, of course, no such settlement was reached. Our analysis suggests that, insofar as Germany and Great Britain did not trade with each other, each side had an interest to fight, even if costly, for TOT (among other) reasons.

[49] In the limiting case where $\sigma = \infty$, this effect vanishes and arming incentives for both adversaries are identical across the two trade regimes.

[50] Furthermore, we expect that admitting the possibility of trade in arms between the third friendly country and each of the two adversaries would not change our results qualitatively and, in fact, could amplify the positive effect of trade on arming incentives, implying even greater security costs and a larger likelihood of negative welfare consequences.

[51] Tariffs render the analysis much more complex because of TOT and volume-of-trade effects.

Proposition 7 (*Differences in non-tariff trade costs with friends and relative arming under symmetry.*) *In a unique trade equilibrium with positive trade flows, differences in non-tariff trade costs across two adversaries that each trade with a third, friendly country generate differences in equilibrium arming as follows: $\tau^i \lesseqqgtr \tau^j$ implies $G^i_T \gtreqqless G^j_T$.*

In sharp contrast to the case where two adversaries trade with each other, the country facing lower trade costs tends to arm by more.[52] Numerical analysis confirms this result holds beyond the case of identical adversaries. What's more, if τ^j is fixed, then a decrease in τ^i induces greater arming by both adversaries.[53] We have also studied numerically an extension of the model to three goods. Maintaining our assumption that the adversaries $(i = 1, 2)$ are identical, but assuming each country $i = 1, 2, 3$ enjoys a comparative advantage in producing good i, we suppose that (i) $\tau^i_j = \tau^j_i$ and (ii) $\tau^3_i = \tau^i_3$ for $i \neq j = 1, 2$. Then, we find trade-cost reductions between adversaries reduce arming, whereas trade-cost reductions between friends expand arming: $dG^i_T/d\tau^i_j > 0$ and $dG^i_T/d\tau^3_i < 0$ for $i \neq j = 1, 2$.[54]

6 Empirical Evidence: Trade Costs and Military Spending

Our theory supplemented with numerical analysis generates testable predictions regarding the impact of trade costs on military spending. Specifically, consistent with the classical liberal view, the analysis in Section 4 implies that trade costs between rivals should have a positive impact on their military spending; by contrast, the analysis of Section 5 suggests that trade costs between countries viewed as friends could have a negative effect on their military spending, provided they have rivals.

6.1 Empirical Specification and Strategy

To test these predictions, we specify the following econometric model:

$$\begin{aligned} \mathit{MLTRY\_SPEND}_{i,t} \;&=\; \beta_0 + \beta_1 \mathit{TRADE\_COSTS\_RIVALS}_{i,t} + \beta_2 \mathit{TRADE\_COSTS\_FRIENDS}_{i,t} \\ &\quad + \beta_3 \mathit{TRADE\_COSTS\_NR}_{i,t} + \mathbf{CONTROLS_{i,t}}\boldsymbol{\beta_4} + \xi_t + \xi_i + \epsilon_{i,t}, \quad (17) \end{aligned}$$

where $\mathit{MLTRY\_SPEND}_{i,t}$ is the logarithm of military expenditures in country i at time t, taken from the Stockholm International Peace Research Institute (SIPRI). The two key covariates of interest are $\mathit{TRADE\_COSTS\_RIVALS}_{i,t}$ and $\mathit{TRADE\_COSTS\_FRIENDS}_{i,t}$, reflecting the trade costs of countries that have rivals, classified as such using data on "strate-

[52] Of course, in the case where $\sigma = 1$, expenditure shares are independent of prices and thus trade costs, such that $G^i_T = G^j_T$ even when $\tau^i \neq \tau^j$.

[53] In addition, a decrease in τ^i for given τ^j increases V^i_T and reduces V^j_T. However, we also find that, when bilateral trade costs are symmetric (i.e., $\tau^i = \tau^3_i$ for $i = 1, 2$), $\tau^i \lesseqqgtr \tau^j$ implies $G^i_T \lesseqqgtr G^j_T$ due to a differential (and dominant) effect on import shares. (See Supplementary Appendix B.2 for more details.) Nonetheless, numerical analysis shows that, for fixed $\tau^j = \tau^3_j$, an increase in $\tau^i = \tau^3_i$ leads to a decrease in arming by both adversaries provided that τ^i is sufficiently high.

[54] Details are available from the authors on request.

gic rivalries" between country-pairs from Thompson (2001).[55] $TRADE\_COSTS\_RIVALS_{i,t}$, in particular, is defined as the logarithm of the weighted average of the bilateral trade costs faced by country i for exports to its rivals at time t, with the size of the rival's market used as the weight.[56] Our theory predicts $\beta_1 > 0$. $TRADE\_COSTS\_FRIENDS_{i,t}$ is defined as the logarithm of the weighted average of the bilateral trade costs faced by country i (which has rivals) for exports to its friends. According to our theory, we expect $\beta_2 < 0$.

In addition to the theoretically motivated covariates in (17), we also control for other potential determinants of military spending. In particular, since only a subset of the countries in our sample have rivals, we add $TRADE\_COSTS\_NR_{i,t}$, defined as the logarithm of the weighted average of trade costs faced by the countries having no rivals in our sample. The vector $\mathbf{CONTROLS_{i,t}}$ includes proxies for country size, such as the logarithm of Gross Domestic Product ($GDP_{i,t}$) and the logarithm of i's population ($PPLN_{i,t}$). We also control for the stability of a country's government ($INSTITUTIONS_{i,t}$), for whether a country has rivals or not ($RIVALS_{i,t}$), and for the extent to which the country is involved in geopolitical conflict ($HOSTILITY_{i,t}$).[57] In the spirit of Redding and Venables (2004), we control further for relative size/market power within rivalries using the following index:

$$RLTV\_POWER_{i,t} \equiv \ln\left(Y_{i,t} \times \Pi_{i,t}^{\sigma-1} / \left[\sum\nolimits_j E_{j,t} \times P_{j,t}^{\sigma-1}\right]\right). \tag{18}$$

The numerator in (18) captures the market power of the exporters in country i in time t, and it is defined as the product between the value of output in i, $Y_{i,t}$, and the outward multilateral resistance, $\Pi_{i,t}^{\sigma-1}$, of Anderson and van Wincoop (2003), which can be interpreted as a "Market Access" index following Redding and Venables (2004). Similarly, the denominator in (18) is designed to capture the market power of importers in the destination/rival country j, and it is defined as the product between the size of the rival's market j, measured

[55] More precisely, Thompson's (2001) indicator records whether or not there exists a threat of war between two countries based on observed diplomatic tensions; as such, two countries need not be involved in a war to be classified as "strategic rivals." We view Thompson's definition of strategic rivalry, which does not rely exclusively on militarized disputes, as particularly appropriate for our purposes. Furthermore, Thompson's dataset includes the largest number of strategic rivalry cases, as compared with Diehl and Goertz (2000) and Bennett (1996, 1997). Supplementary Appendix B.3 offers some details regarding these data and the countries covered in our sample.

[56] We follow the structural gravity literature to construct our trade-cost variables, using data taken from Baier et al. (2019) that are based on total manufacturing trade data from UNCTAD's COMTRADE. First, we apply the latest developments in the empirical gravity literature (Head and Mayer, 2014) to obtain the vector of bilateral trade costs from a panel Poisson Pseudo-Maximum-Likelihood (PPML) specification with exporter-time, importer-time and pair fixed effects, along with time-varying bilateral policy variables for free trade agreements (FTAs), membership to the World Trade Organization (WTO), and a set of time-varying border variables to account for common globalization effects. Guided by the theoretical gravity literature (e.g., Anderson and Neary, 2005; Costinot and Rodríguez-Clare, 2014), we then aggregate bilateral trade costs to the country level. See Supplementary Appendix B.3 for further details.

[57] Data on these additional control variables come from the dynamic gravity data set of the U.S. International Trade Commission. (See Gurevich and Herman (2018) for details on this database.)

by j's expenditure $E_{j,t}$, and the inward multilateral resistance, $P_{j,t}^{\sigma-1}$, of Anderson and van Wincoop (2003), which can be interpreted as a "Supplier Access" index following Redding and Venables (2004).[58] The summation in the denominator of (18) is used to deal with the few instances where a given country has more than one rival at the same time.[59]

Finally, in most of our specifications, we include year fixed effects (ξ_t) to capture any global trends. In our most demanding specification, we also employ country fixed effects (ξ_i) to account for any observable and unobservable time-invariant country characteristics that could affect military spending. Combining all data sources resulted in an unbalanced panel dataset with 67 countries over the period 1986–1999. The starting date of our sample was determined by the trade dataset of Baier et al. (2019) used to construct the trade-cost variables, while the ending date was set by Thompson's (2001) strategic rivals dataset.[60]

6.2 Results

Our main estimates appear in Table 1. To establish the robustness of our findings and to better understand the channels through which trade costs affect military expenditures, we start with simple correlations on which we build gradually. Column (1) of Table 1 reports the results when we include only the three trade-cost covariates. The estimates of the effects of $TRADE\_COSTS\_RIVALS_{i,t}$ and $TRADE\_COSTS\_FRIENDS_{i,t}$ are precisely as predicted by our theory. Specifically, the positive estimate of β_1 suggests that greater trade costs with rivals weaken the pacifying effects of trade on their military spending, while the negative estimate of β_2 suggests that, for countries having rivals in our sample, greater trade costs with friends weaken the amplifying effects of trade on their the military spending. Finally, the large, negative, and statistically significant estimate of the coefficient on $TRADE\_COSTS\_NR_{i,t}$ reflects an inverse relationship between trade costs and military spending for countries having no rivals in our sample. While our theory does not speak directly to this last result, it appears to be fairly robust and strikes our interest as it seems to contradict the classical liberal view.

Notably, the R^2 of the estimated model in column (1), consisting of only three covariates, seems quite large. In general, and especially in combination with the varying signs, the good fit suggests a strong correlation between trade costs and military expenditures, which is encouraging for this line of research. One possible explanation for the strong fit is that the trade-cost measures proxy for national trade, which in turn is highly correlated with country

[58] Capitalizing on the additive property of the PPML estimator (see Arvis and Shephard, 2013; Fally, 2015) applied to the same specification that we use to estimate bilateral trade costs (equation (B.26) presented in the Supplementary Appendix B.3), we recover the numerator in (18) from the exporter-time fixed effects in our estimating gravity model as $Y_{i,t} \times \Pi_{i,t}^{\sigma-1} = \chi_{i,t}$. Similarly, we obtain the denominator in (18) from the importer-time fixed effects in the same gravity regression as $E_{j,t} \times P_{j,t}^{\sigma-1} = \varphi_{j,t}$.

[59] Such instances arise in our sample for Argentina, Egypt, and Israel.

[60] Additional details regarding the countries and active rivalries between them in our sample, as well as summary statistics, can be found in Supplementary Appendix B.3.

size, and larger countries tend to devote more resources to military spending. However, as we demonstrate next, even in the presence of the full set of controls, the trade-cost covariates remain significant and with the predicted signs.

Building on our baseline model, we introduce additional control variables and add year fixed effects in column (2) of Table 1. The estimated coefficients on the control variables are mostly intuitive. Specifically, the positive estimates of the effects of $GDP_{i,t}$ suggests that larger countries devote more resources to military spending. The negative and statistically significant estimates of the coefficients on $PPLN_{i,t}$ and $INSTITUTIONS_{i,t}$ show that, all else equal, countries with larger population and stronger institutions devote less resources to their respective military sectors. The positive estimate of the effects of $HOSTILITY_{i,t}$ is also intuitive, suggesting that countries with more hostile environments devote more resources to their militaries. The estimate of the coefficient on $RIVALS_{i,t}$ is negative, but not statistically significant. This finding and the statistical insignificance of the estimate on $RLTV\_POWER_{i,t}$ suggest the other control variables successfully control for the variation in these covariates in our specification. Most importantly for our purposes, the estimates of the coefficients on the two key covariates for trade costs with friends and rivals, though smaller in magnitude, retain their signs and remain statistically significant.

Arguably, our focus on trade costs in this empirical analysis circumvents a number of endogeneity concerns that would arise had we used trade flows instead. Specifically our approach, by design, removes all country-specific determinants of trade flows. Nevertheless, some endogeneity concerns could remain. One such possibility is omitted variable bias. For example, the indicator variable for rivalries in the Thompson data fails to capture likely variation in the intensity of ongoing rivalries between countries over time that could influence both their defense expenditures and their trade policies, and thus their trade costs. Other possible concerns include the general equilibrium forces and the endogeneity of trade policies that can impact the weights used to aggregate bilateral trade costs.

To address these issues, we implement an IV estimator. Specifically, we follow Feyrer (2009, 2019) to construct an instrument that relies on relative changes in distance-based trade costs as[61]

$$predicted\ trade_{i,t} = \exp\left(\hat{\kappa}_t\right) \sum_{i \neq j} \exp\left(\hat{\kappa}_{ij} + \hat{\beta}_{dist,t} DIST_{ij}\right), \tag{19}$$

where, $\hat{\kappa}_t$ and $\hat{\kappa}_{ij}$ are the estimates of the time fixed effects and the country pair fixed effects from a bilateral gravity regression; $\hat{\beta}_{dist,t}$ are the time-varying estimates of the impact of distance on trade from the same regression. Given our setting, we construct two such

[61] Consistent with our approach of employing pair fixed effects to model bilateral trade costs in our structural gravity estimations, we implement the more demanding specification to construct the instrument from Feyrer with pair fixed effects. This is equation (12) on page 15 in Feyrer (2019).

instruments, one for trade with friends and one for trade with rivals. For both, we employ the distance between the capital cities.[62]

The estimates reported in column (3) of Table 1 are obtained with an IV estimator that employs the two instruments just described. We also include lags of the two instruments that allow us to gauge their validity via an over-identification test. Two findings stand out. First, our instruments pass the "weak identification" (WeakId) Kleibergen-Paap Wald test for joint significance of the instruments as well as the joint over-identification test with $\chi^2 = 1.482$, (p-$val = 0.477$). Second and more importantly, the estimated effects of the two key trade variables remain statistically significant and have the predicted signs.

We conclude the analysis with our most demanding specification where, in addition to using the IV estimator from column (3), we introduce country fixed effects and account for clustering of observations at the country level. Several findings stand out from the resulting estimates reported in column (4). First, a number of the covariates lose statistical significance; some (for example, $TRADE\_COSTS\_FRIENDS_{i,t}$, $GDP_{i,t}$, and $PPLN_{i,t}$) are no longer statistically significant. We attribute this result largely to the inclusion of the country fixed effects. In particular, since country fixed effects account for and absorb all time-invariant country-specific variables that influence military spending, we are left only with variation over time within countries to identify the impact of the country-specific covariates in our setting; that we also include fixed year effects and our sample covers a relatively sort time period reinforces this effect.[63] Second, we see in column (4) that, even in the presence of the country fixed effects and the clustered standard errors, the estimate on $TRADE\_COSTS\_RIVALS_{i,t}$ remains sizable, marginally statistically significant, and with a positive sign as predicted by our theory. In addition, the estimate on $TRADE\_COSTS\_FRIENDS_{i,t}$, though statistically insignificant, has a negative sign as predicted and its magnitude has not changed much either.[64] Finally, the last column of

[62] This measure of distance differs from the two used by Feyrer (2019): (i) the standard population-weighted distance measure from the Centre d'Études Prospectives et d'Informations Internationales (CEPII) and (ii) the measure of sea distance, constructed by Feyrer himself. We do not use Feyrer's sea-distance measure since the match between the country pairs covered by this measure and the countries in our sample leads to a substantial loss in the number of observations. We do not use the standard population-weighted distance either, because it might not be fully exogenous in our setting, particularly if there are territorial conflicts that alter borders and overall distance between countries. However, even in such cases, it seems unlikely that the distance between capital cities would be affected. It should also be noted that the correlations between the three distance measures (sea, population-weighted and between capitals) is larger than 0.9.

[63] Let us add, however, that the inclusion of the country fixed effects and the clustering of standard errors by country appear to have independent effects. Specifically, when we introduce these estimation features sequentially, we find that the country fixed effects have a greater impact on the estimates and corresponding standard errors on $GDP_{i,t}$ and $PPLN_{i,t}$, while clustering the standard errors per country has a greater impact on the significance of the trade-cost variables.

[64] We also perform three sensitivity experiments that confirm the robustness of our findings: (i) we use a vector of bilateral trade costs based only on estimates of the pair fixed effects from a structural gravity regression; (ii) we employ theory-consistent weights to aggregate the vector of bilateral trade costs to the country level; and, (iii) we obtain estimates using only years from our sample that correspond to the Cold

Table 1 reports standardized (beta) coefficient estimates, which reveal the large relative importance of the estimates of the two key trade costs covariates in support of our theory.

7 Concluding Remarks

This paper develops a Ricardian model of trade, modified to include two primary resources and augmented by conflict between two countries over one of those resources. It features a TOT channel that renders arming choices trade-regime dependent. Specifically, a country's arming under trade has an additional effect on its own payoff, by influencing the adversary's production and thus world prices. Exactly how trade influences arming incentives depends on the structure of comparative advantage and trade costs.

When the two countries in conflict also trade with each other, the impact of a country's arming on its TOT is negative. Provided these countries are sufficiently symmetric, not only in terms of technologies and preferences, but also in terms of the mix of their secure resource endowments, equilibrium arming by both is lower and their payoffs higher under trade than under autarky. These results, which are robust to the presence of trade costs, provide theoretical support to the long-standing classical liberal hypothesis that increased trade openness can ameliorate conflict and thus amplify the gains from trade. With sufficiently extreme differences in the distribution of the primary resources, a shift to trade could induce one country to arm more heavily and to such an extent so as to imply that autarky is preferable over trade to the other country.[65] Nevertheless, in an equilibrium that involves positive trade flows, the aggregate allocation of resources to dispute the insecure resource is lower than in an equilibrium with no trade at all.

When the structure of comparative advantage is such that the two adversaries do not trade with each other, but instead trade with a third, friendly country and they compete in the same export market, the TOT effect of security policies is positive. As such, a shift from autarky to trade unambiguously intensifies international conflict, possibly with negative net welfare consequences.

Consistent with the model's predictions, our empirical analysis provides reduced-form evidence that the effects of trade costs on a country's military spending depend qualitatively on whether trade is with a rival or with a friend. Our findings complement the more structural evidence presented by Martin et al.'s (2008), that increased opportunities for multilateral trade can aggravate bilateral conflict, increasing the likelihood of war. They also complement Seitz et al. (2015)'s evidence that a decrease in trade costs between two countries reduces their military spending, which reduces such spending by other countries.

War period. See Supplementary Appendix B.3 for further details.

[65] In ongoing research, we have extended the the analysis to capture the presence of a non-tradable goods sector, finding that, even in the case of complete symmetry, different factor intensities across the tradable and non-tradable sectors can cause the classical liberal view to fail.

A potentially fruitful extension of our analysis involves considering multiple commodities traded among multiple trading partners (for example, along the lines of Eaton and Kortum, 2002). While capturing the direct and cross-price effects of security policies, this modeling choice would enable us to expand the sets of countries to explore, for example, the theoretical implications of trade among countries with no rivals for arming. We could, then, scrutinize our finding of a negative and fairly robust correlation between trade costs and arming for such countries. This extension could also incorporate the presence of nontraded sectors, thereby mapping more closely to reality. Equally important, by admitting the presence of multiple channels of influence between trade and arming, this richer setting would be more suitable for structural estimation. As such, it could complement nontrivially the analysis of Acemoglu and Yared (2010), who emphasize the effects of national military spending on trade, by studying the two-way causality between arming and trade.

The analysis could also be extended to study how security policies matter for trade agreements by conditioning trade negotiations on a threat-point that coincides with the non-cooperative Nash equilibrium in trade and security policies. This approach could also shed light on the implications of linking security policies, not just with trade agreements, but also with sanctions and foreign aid (Maggi, 2016).

Acknowledgments

This is a substantially revised version of a paper that circulated under the title of "Trading with the Enemy." We are heavily indebted to Stergios Skaperdas for extensive discussions, as well as to Costas Arkolakis and two anonymous referees for extremely useful comments. We also thank Mostafa Beshkar, Rick Bond, James Feyrer, Kamal Saggi, Jun Xiang, Ben Zissimos and participants of the ASSA Meetings 2015, the WEAI Annual Conference 2015, the Midwest Economic Theory Conference at Purdue University 2016, the International Economics Workshop at Vanderbilt University and the UC-Irvine/GPACS Workshop on Economic Interdependence and War. Oscar Camacho provided excellent research assistance.

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Table 1: International Trade Costs and Military Spending

	(1)	(2)	(3)	(4)	(5)
	CORR	CNTRLS	IV	IV-FEs	BETAs
TRADE_COSTS_RIVALS	0.229	0.103	0.214	0.202	0.370
	(0.039)**	(0.052)*	(0.082)**	(0.116)$^{+}$	
TRADE_COSTS_FRIENDS	-0.775	-0.082	-0.226	-0.220	-0.460
	(0.028)**	(0.038)*	(0.065)**	(0.576)	
TRADE_COSTS_NR	-0.666	-0.113	-0.152	-0.085	-0.185
	(0.023)**	(0.032)**	(0.036)**	(0.557)	
LN_GDP		1.461	1.347	0.059	0.042
		(0.062)**	(0.068)**	(0.272)	
LN_PPLN		-0.546	-0.507	0.403	0.267
		(0.042)**	(0.044)**	(0.673)	
INSTITUTIONS		-0.053	-0.053	-0.032	-0.093
		(0.008)**	(0.009)**	(0.010)**	
HOSTILE		2.751	2.525	1.051	0.028
		(0.484)**	(0.441)**	(0.243)**	
RIVALS		-0.510	-0.379	-0.253	-0.049
		(0.313)	(0.416)	(0.358)	
RLTV_POWER		0.165	0.234	0.078	0.023
		(0.114)	(0.129)$^{+}$	(0.211)	
Year Fixed Effects	No	Yes	Yes	Yes	Yes
Country Fixed Effects	No	No	No	Yes	Yes
N	904	875	810	810	810
R^2	0.647	0.807			
Weak Id χ^2			65.793	11.04	
Over Id χ^2			1.456	0.695	

Notes: This table reports results from a series of specifications, based on equation (17), that quantify the impact of trade costs on military spending. The dependent variable is always the logarithm on national military spending and all estimates are obtained with the OLS estimator. All specifications distinguish between the impact of trade costs of countries that have rivals to their rivals (*TRADE_COSTS_RIVALS*) or to their friends (*TRADE_COSTS_FRIENDS*). The estimates in column (1) are correlations obtained without any controls and without any fixed effects. The estimates in column (2) introduce additional controls, as described in the main text, and add year fixed effects. The estimates in column (3) are obtained with an IV estimator. The results in column (4) are obtained with the IV specification from column (3) but after also adding a full set of country fixed effects. Finally, the last column reports standardized (beta) coefficients, which are based on the specification in column (4). Columns (1)-(3) report robust standard errors, while the estimates in column (4) are obtained with standard errors clustered by country. $^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$. See the main text for further details.

A Appendix

This appendix presents proofs of the propositions and lemmas in the main text. Additional results and more technical details can be found in Supplementary Appendix B.

Lemma A.1 *For any given feasible quantities of guns and gross factor endowments, the equilibrium wage-rental ratio ω^i is independent of the prevailing trade regime (autarky or trade). Furthermore,*

(a) $\partial Z^i(\omega^i, \cdot)/\partial \omega^i = 0$ *and* $\partial^2 Z^i(\omega^i, \cdot)/\partial(\omega^i)^2 > 0$*, s.t.* $\omega^i = \arg\min_{\omega^i} Z^i(\omega^i, \cdot)$;

(b) $\partial \omega^i/\partial K_g^i > 0$, $\partial \omega^i/\partial L_g^i < 0$*, and* $\partial \omega^i/\partial G^i \gtreqless 0$ *if* $c_r^i/c_w^i \gtreqless \psi_r^i/\psi_w^i$;

(c) $\partial Z^i/\partial K_g^i > 0$ *and* $\partial^2 Z^i/(\partial K_g^i)^2 < 0$;

(d) $\partial Z^i/\partial L_g^i > 0$ *and* $\partial^2 Z^i/(\partial L_g^i)^2 < 0$;

(e) $\partial Z^i/\partial G^i < 0$ *and* $\partial^2 Z^i/(\partial G^i)^2 < 0$ *for fixed* K_g^i.

Proof: See Supplementary Appendix B.1.

Proof of Proposition 1. Let $\overline{G}^i$ be the quantity of guns country i would produce if all of L^i and K^i were employed in that sector. Assuming $f(0)$ is arbitrarily close to 0 implies that $\lim_{G^i \to 0} f'(G^i)$ and thus $\lim_{G^i \to 0} \phi_{G^i}^i$ in (11) are arbitrarily large for any $G^j \geq 0$. Therefore, $\partial V_A^i/\partial G^i > 0$ as $G^i \to 0$. Furthermore, by the definition of $\overline{G}^i$ and our assumption that both factors are essential in the production of Z^i, $V_A^i(\overline{G}^i, G^j) < V_A^i(G^i, G^j)$ for all $G^i < \overline{G}^i$ that imply $Z^i > 0$; therefore, $\partial V_A^i/\partial G^i < 0$ for a sufficiently large $G^i \in [0, \overline{G}^i]$. The continuity of V_A^i in G^i, then, implies that there exists a best-response function for each country i, $G^i = B_A^i(G^j) \in (0, \overline{G}^i)$, such that (11) holds as an equality.

Existence. As in Garfinkel et al. (2015), to establish existence it suffices to show $\partial^2 V_A^i / \left(\partial G^i\right)^2 < 0$ at $G^i = B_A^i(G^j)$. In the proof of Lemma A.1, we establish the following effects of marginal changes in w^i, K_g^i and G^i on Z_w^i:

$$
\begin{aligned}
Z_{ww}^i &= -(c_{ww}^i Z^i + \psi_{ww}^i G^i)/c^i > 0 \\
Z_{wK}^i &= -r^i c_w^i/(c^i)^2 < 0 \\
Z_{wG^i}^i &= -(\psi_w^i c^i - c_w^i \psi^i)/(c^i)^2.
\end{aligned}
$$

Then, while keeping r^i fixed in the background (thus attributing any implied changes in ω^i to changes in w^i) and using the FOC associated with B_A^i in (11), an application of the implicit function theorem to the envelope condition $Z_\omega^i(\omega^i, \cdot) = 0$ shows:

$$
w_{G^i}^i\big|_{G^i = B_A^i} = -\frac{K_0 Z_{wK}^i \phi_{G^i}^i + Z_{wG^i}^i}{Z_{ww}^i} = -\frac{\psi_w^i}{c_{ww}^i Z^i + \psi_{ww}^i G^i} > 0. \qquad \text{(A.1)}
$$

Since $\psi_w^i > 0$, the concavity of the unit cost functions in factor prices (i.e., $c_{ww}^i, \psi_{ww}^i < 0$) implies, in turn, that an increase in G^i in the neighborhood of B_A^i increases the country's market-clearing (relative) wage, regardless of factor intensities.

Differentiating (11) with respect to G^i, after simplifying and using (A.1), shows

$$\left.\frac{\partial^2 V_A^i}{(\partial G^i)^2}\right|_{G^i=B_A^i} = \frac{m_A^i}{c^i}\left[r^i K_0 \phi^i_{G^iG^i} + \frac{\left(\psi_w^i\right)^2}{c^i_{ww}Z^i + \psi^i_{ww}G^i}\right] < 0, \tag{A.2}$$

where we now define $m_A^i \equiv m(p_A^i)$. The negative sign of (A.2) follows from the concavity of the conflict technology in G^i (i.e., $\phi^i_{G^iG^i} < 0$) and equation (A.1). The strict quasi-concavity of V_A^i in G^i, in turn, implies the existence of an interior equilibrium.

Uniqueness. To prove uniqueness of equilibrium, it suffices to show that

$$|J| \equiv \frac{\partial^2 V_A^i}{(\partial G^i)^2}\frac{\partial^2 V_A^j}{(\partial G^j)^2} - \frac{\partial^2 V_A^i}{\partial G^i \partial G^j}\frac{\partial^2 V_A^j}{\partial G^j \partial G^i} > 0, \tag{A.3}$$

at any equilibrium point or, equivalently,

$$\frac{\partial B_A^i}{\partial G^j}\frac{\partial B_A^j}{\partial G^i} < 1, \text{ where } \quad \frac{\partial B_A^i}{\partial G^j} = -\frac{\partial^2 V_A^i}{\partial G^i \partial G^j} \bigg/ \frac{\partial^2 V_A^i}{\left(\partial G^i\right)^2} \quad \text{for } i \neq j = 1, 2.$$

From the existence part of this proof, the sign of $\partial B_A^i/\partial G^j$ is determined by the sign of $\partial^2 V_A^i/\partial G^i \partial G^j$. To proceed, apply the implicit function theorem to $Z_\omega^i(\omega^i, \cdot) = 0$ to find

$$w^i_{G^j}\big|_{G^i=B_A^i} = -\frac{K_0 Z^i_{wK}\phi^i_{G^j}}{Z^i_{ww}} = -\frac{r^i K_0 \phi^i_{G^j} c^i_w}{c^i(c^i_{ww}Z^i + \psi^i_{ww}G^i)} < 0, \tag{A.4}$$

where the negative sign follows from the facts that $c^i_w > 0$, $c^i_{ww}, \psi^i_{ww} < 0$, and $\phi^i_{G^j} < 0$. Differentiating (11) with respect to G^j, while using (A.4), gives

$$\left.\frac{\partial^2 V_A^i}{\partial G^i \partial G^j}\right|_{G^i=B_A^i} = \frac{m_A^i}{c^i}\left[r^i K_0 \phi^i_{G^iG^j} + \frac{r^i K_0 \phi^i_{G^j} c^i_w \psi^i_w}{c^i(c^i_{ww}Z^i + \psi^i_{ww}G^i)}\right]. \tag{A.5}$$

The second term inside the square brackets is positive by (A.4). From (5), the first term is non-negative for all $B_A^i(G^j) \geq G^j$, implying that the expression above is positive, and thus G^i depends positively on G^j. But, G^i will depend negatively on G^j, if $B_A^i(G^j)$ is sufficiently smaller than G^j. Thus, depending on fundamentals, two possibilities arise: (i) $\partial B_A^i/\partial G^j < 0$ and $\partial B_A^j/\partial G^i \geq 0$ for $i \neq j = 1, 2$; or (ii) $\partial B_A^i/\partial G^j > 0$ for both $i \neq j = 1, 2$. However, $|J| > 0$ in case (i). Hence, we need to consider only case (ii).

As in Jones (1965), denote the cost shares of K^i and L^i in producing Z^i respectively by $\theta^i_{KZ} \equiv r^i c^i_r/c^i$ and $\theta^i_{LZ} \equiv w^i c^i_w/c^i$. Similarly let $\theta^i_{KG} \equiv r^i \psi^i_r/\psi^i$ and $\theta^i_{LG} \equiv w^i \psi^i_w/\psi^i$ denote the corresponding cost shares in the production of G^i. In addition, let $\sigma^i_Z \equiv c^i c^i_{wr}/c^i_w c^i_r$ and $\sigma^i_G \equiv \psi^i \psi^i_{wr}/\psi^i_w \psi^i_r$ be the elasticities of substitution between factor inputs in Z^i and

G^i respectively, and define

$$\lambda^i \equiv \frac{\theta^i_{LG}\psi^i G^i}{\theta^i_{LG}\psi^i G^i \theta^i_{KG}\sigma^i_G + \theta^i_{LZ} c^i Z^i \theta^i_{KZ}\sigma^i_Z} > 0. \tag{A.6}$$

Applying the above definitions and country i's FOC (11), while using the linear homogeneity of c^i and ψ^i, allows us to rewrite (A.2) and (A.5) respectively as

$$\left.\frac{\partial^2 V^i_A}{(\partial G^i)^2}\right|_{G^i=B^i_A} = -\frac{m^i_A \psi^i \phi^i_{G^i}}{c^i \phi^i}\left[-\frac{\phi^i \phi^i_{G^i G^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LG}\right] \tag{A.7a}$$

$$\left.\frac{\partial^2 V^i_A}{\partial G^i \partial G^j}\right|_{G^i=B^i_A} = -\frac{m^i_A \psi^i \phi^i_{G^j}}{c^i \phi^i}\left[-\frac{\phi^i \phi^i_{G^i G^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LZ}\right]. \tag{A.7b}$$

Combining these equations gives $\partial B^i_A/\partial G^j = (-\phi^i_{G^j}/\phi^i_{G^i})H^i$, where

$$H^i \equiv \frac{H^i_1}{H^i_2} \equiv \left[-\frac{\phi^i \phi^i_{G^i G^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LZ}\right] \Bigg/ \left[-\frac{\phi^i \phi^i_{G^i G^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LG}\right]. \tag{A.8}$$

Since $(-\phi^i_{G^j}/\phi^i_{G^i})(-\phi^j_{G^i}/\phi^j_{G^j}) = 1$, we have $(\partial B^i_A/\partial G^j)(\partial B^j_A/\partial G^i) = H^i H^j$. Given our focus on the case of strategic complements (which implies $H^i > 0$ for $i = 1, 2$), proving $|J| > 0$ requires only that $H^i < 1$. In Supplementary Appendix B.1, we show that a sufficient (but hardly necessary) condition is that σ^i_G and/or σ^i_Z are large enough. $\|$

Proof of Proposition 2. In the absence of trade costs, country i's TOT coincides with its domestic relative price p^i_T, where $p^i_T \in \left[1/\alpha^j, \alpha^i\right]$. Our proof to follow abstracts from the boundary conditions on p^i_T. As such, it demonstrates the existence only of a local optimum for each country i, given G^j.[66] In Supplementary Appendix B.1, we return to this issue, to show how weak comparative advantage (i.e., low values of α^i) matters and identify the existence of a critical value of α^i for each country i, denoted by α^i_{0T} (> 1), such that $\alpha^i > \alpha^i_{0T}$ for both i ensures that the boundary constraints on p^i_T do not bind in the equilibrium identified here.

Let $\widetilde{V}^i_T(G^i, G^j)$ denote the unconstrained value function for country i under free trade (i.e., abstracting from the limits on p^i_T), and recall our definition of $\overline{G}^i$, as the maximum quantity of arms produced by country i using all of K^i and L^i. Assuming both labor and capital are essential in producing Z^i, $\widetilde{V}^i_T(\overline{G}^i, G^j) < \widetilde{V}^i_T(G^i, G^j)$ for all $G^i < \overline{G}^i$ that imply $Z^i > 0$. Therefore, $\partial \widetilde{V}^i_T/\partial G^i < 0$ for sufficiently large $G^i \in [0, \overline{G}^i]$. If $\sigma \leq \overline{\sigma}^i \equiv \gamma_j/\zeta^j$ evaluated at $G^i = G^j = 0$ so that the marginal benefit of arming is non-positive at $G^i = 0$,

[66] Our abstraction can be thought of as assuming that $\alpha^i \to \infty$ for $i = 1, 2$, which effectively reduces the model to one in which each country produces a nationally differentiated good, as in Armington (1969). Although convenient for ruling out possible discontinuities in the best-response functions, this simplification fails to capture the rich welfare implications we identify in our modified Ricardian model.

the conflict technology (5) implies $\widetilde{V}_T^i(G^i, G^j)$ reaches a peak in the domain $[0, \overline{G}^i]$ at $G^i = 0$ for $G^j \geq 0$.[67] Alternatively, if $\sigma > \overline{\sigma}^i$ so that the marginal benefit of arming is strictly positive at $G^i = 0$, then $\widetilde{V}_T^i(G^i, G^j)$ reaches a peak in the interior of the domain. In this case, equation (5) and the continuity of $\widetilde{V}_T^i(G^i, G^j)$ in G^i imply the existence of an unconstrained best-response function for country i (i.e., ignoring the constraints on p_T^i), denoted by $\widetilde{B}_T^i(G^j)$, such that the FOC (13) holds with equality.

Without any loss of generality, assume that $\overline{\sigma}^1 < \overline{\sigma}^2$. Then, since the above holds true for both countries $i = 1, 2$, we have three possibilities to consider:

(i) $\sigma \leq \overline{\sigma}^1$, which implies $\partial \widetilde{V}_T^i / \partial G^i|_{G^i=0} < 0$ for $i = 1, 2$;
(ii) $\sigma \in (\overline{\sigma}^1, \overline{\sigma}^2]$ which implies $\partial \widetilde{V}_T^1 / \partial G^1|_{G^1=0} < 0$, while $\partial \widetilde{V}_T^2 / \partial G^2|_{G^2=0} > 0$;
(iii) $\sigma > \overline{\sigma}^2$, which implies $\partial \widetilde{V}_T^i / \partial G^i|_{G^i=0} > 0$ for $i = 1, 2$.

In case (i), there exists an equilibrium where each country i chooses $G_T^i = 0$. Similarly, $G_T^i = 0$ in case (ii) for country i that has $\partial \widetilde{V}_T^i / \partial G^i < 0$ at $G^i = 0$. By contrast, in case (ii) for the other country $j \neq i$ and in case (iii) for both countries i, the marginal net benefit from producing arms, when evaluated at zero arming, is strictly positive. Hence, to establish the existence of a pure-strategy equilibrium in security policies under trade $(G_T^1, G_T^2) \neq (G_A^1, G_A^2)$, assuming that α^i is arbitrarily large for both i, we need to prove only that $\widetilde{V}_T^i(G^i, G^j)$ is strictly quasi-concave in G^i for country i when $\partial \widetilde{V}_T^i / \partial G^i = 0$ at $G^i = \widetilde{B}_T^i(G^j) > 0$ given $G^j \geq 0$, in cases (ii) and (iii).[68] We present this part of the proof in Supplementary Appendix B.1. ||

Proof of Lemma 2. We decompose the payoff effects of a shift from autarky to free trade into three parts. First, starting from the equilibrium under autarky, allow country j to reduce its guns from G_A^j to $G_T^j \geq 0$, and let G^i adjust along $B_A^i(G^j)$, so that only the strategic effect on country i's payoff matters:

$$\frac{1}{V_A^i} \frac{\partial V_A^i}{\partial(-G^j)}\bigg|_{G^i = B_A^i(G^j)} = -\frac{dZ^i/dG^j}{Z^i} > 0,$$

which implies $V_A^i(B_A^i(G_T^j), G_T^j) > V_A^i(G_A^i, G_A^j)$. Second, consider the welfare effect of a shift to trade, given $G^i = B_A^i(G_T^j)$ and $G^j = G_T^j$. From Lemma 1, such a shift implies a non-negative welfare effect: $V_T^i(B_A^i(G_T^j), G_T^j) \geq V_A^i(B_A^i(G_T^j), G_T^j)$. Third, country i's shift to trade induces it to adjust its arming, from $B_A^i(G^j)$ to $B_T^i(G^j)$ given $G^j = G_T^j \geq 0$. Since this adjustment also produces a non-negative welfare effect, we have $V_T^i(B_T^i(G_T^j), G_T^j) \geq$

[67] As suggested in the main text, another local maximum for $G^i \in (0, \overline{G})$ could exist.

[68] Note that, by the continuity of our specification of $\phi^i = \phi(G^i, G^j)$ in (5) at $G^1 = G^2 = 0$, with an appropriate parameterization of this conflict technology (e.g., in terms of the parameters δ and b for the example shown in footnote 22), we can ensure that $\sigma - \gamma_2/\zeta^2$ remains negative when evaluated at country 2's best response to $G^1 = 0$ for case (ii).

$V_T^i(B_A^i(G_T^j), G_T^j)$. Bringing these results together gives $V_T^i(G_T^i, G_T^j) > V_A^i(G_A^i, G_A^j)$ for $i \neq j = 1, 2$. In words, each country's gain with a shift to free trade exceeds that predicted by models that abstract from conflict, as captured by the second part of the decomposition.

What remains to be shown is that each country i's payoff under free trade is comparable to its payoff under autarky for any given $G^j \geq 0$. The proof of Proposition 2 establishes this requirement is satisfied provided that α^i is sufficiently large. Thus, we need to consider only circumstances where the lower price constraint $p_T^i \geq 1/\alpha^j$ binds for country i. Fix G^j at some level, and suppose that $p_T^i = 1/\alpha^j$ for some value of G^i—call it G_c^i. Because the unconstrained optimal value of G^i level is less than G_c^i, we know that $\widetilde{V}_T^i$ and thus V_T^i rise as G^i approaches G_c^i from above. But, V_T^i also rises as G^i approaches G_c^i from below, since the optimal value of G^i at constant prices equals $B_A^i(G^j)$ which exceeds G_c^i, given G^j. Thus, V_T^i reaches a (kinked) peak at G_c^i. ||

Proof of Proposition 3. Here we focus on the sufficient (but not necessary) condition for a shift from autarky to free trade to induce a decrease in arming by both countries—that $\partial B_A^i/\partial G^j > 0$ in the neighborhood of the autarkic equilibrium. Starting with the case of complete symmetry where $L^i = L$ and $K^i = K$ for $i = 1, 2$, suppose the two countries arm identically, $G^1 = G^2 > 0$. Then, from (5), $\phi^i = \frac{1}{2}$ and thus $K_Z^i = K_Z$ for $i = 1, 2$. In turn, (7) and (8) imply $\omega^i = \omega$ and $Z^i = Z$ for $i = 1, 2$. Finally, c^i/r^i and ψ^i/r^i are identical across countries. Taken together, these findings imply the FOCs under autarky (11) are identical across countries. Thus, $G^1 = G^2 > 0$ represents a possible equilibrium. From Proposition 1, we know further that, provided labor and capital are sufficiently substitutable in the production of the intermediate good and guns, the equilibrium $G_A^i = G_A > 0$ for $i = 1, 2$ is unique. Since $\phi_{G^iG^j}^i = 0$ in this case, (A.5) implies our sufficient condition is satisfied.

There exists, in addition, a set of initial distributions of secure resources (including, but not limited to, the symmetric distribution) that similarly imply $G_A^i = G_A > 0$ for $i = 1, 2$. To see this, recall that $k_Z^i \equiv K_Z^i/L_Z^i$ denotes country i' residual capital-labor ratio. Equation (7) shows that $k_Z^i = c_r^i/c_w^i$ determines the equilibrium wage-rental ratio ω_A^i, and consequently $k_{ZA}^i = k_{ZA}$ and $\omega_A^i = \omega_A$ hold for $i = 1, 2$ under complete symmetry. Now, fix each country's guns production at G_A and reallocate both labor and secure capital from country 2 to country 1 such that $dK^i = k_{ZA}dL^i$ for $i = 1, 2$, so as to leave their residual capital-labor ratios unchanged at k_{ZA}. (For future reference, denote the set of the resulting distributions of secure resources by $\mathcal{S}_0$.) But, since k_{ZA} does not change by assumption, ω_A also remains unchanged. By the FOC under autarky (11), then, neither country has an incentive to change its arming with the redistribution of labor and capital. Thus, for asymmetric distributions of secure resources in $\mathcal{S}_0$, equilibrium arming under autarky remains unchanged and equalized across countries. Accordingly, $\partial B_A^i/\partial G^j > 0$

continues to hold for $i \neq j = 1,2$ in the neighborhood of the autarkic equilibrium.[69]

Finally, we consider distributions adjacent to $\mathcal{S}_0$. Starting from any distribution of secure resources in $\mathcal{S}_0$, transfer one unit of labor from country 2 to country 1. By Lemma A.1(b) this sort of transfer for given arming choices increases the relative wage and thus the marginal cost of arming in the donor country (2) and has the opposite effects in the recipient country (1). Thus, by the FOC under autarky (11) along with the strict quasi-concavity of payoffs under autarky demonstrated in the proof to Proposition 1, this transfer of labor induces the recipient (with $k^1_{ZA} \downarrow$) to arm by more and the donor (with $k^2_{KA} \uparrow$) to arm by less, implying $G^1_A > G^2_A$. The logic spelled out above, in turn, implies that country 1's best-response function continues to be positively sloped in the neighborhood of the autarkic equilibrium. Furthermore, for small transfers of labor from country 2 to country 1, country 2's best-response function also remains positively sloped. By continuity, then, $\partial B^i_A/\partial G^j > 0$ continues to hold for $i \neq j = 1,2$ and distributions adjacent to $\mathcal{S}_0$.[70] Hence, provided that the distribution of initial resource endowments across the two countries imply sufficiently similar residual capital-labor ratios, $G^i_T < G^i_A$ holds for $i \neq j = 1,2$.

Proof of Lemma 3. Using (14a) with $\widehat{\tau}^i = \widehat{\tau}^j = \widehat{t}^i = 0$, one can easily verify that the following shows country i's incentive to arm under trade given his rival's arming G^j:

$$\frac{\partial V^i_T/\partial G^i}{V^i_T} = \left(1-\rho^i\right)\frac{dZ^i/dG^i}{Z^i} + \rho^i \frac{dZ^j/dG^i}{Z^j},$$

where as previously defined, $\rho^i \equiv (\gamma^i_j/t^i\Delta)[\left(t^i-1\right)\varepsilon^i + 1]$. Maintaining focus on the case where $\sigma > 1$, we have $\rho^i \in (0,1)$. Next evaluate the above at the solution for arming by country i under autarky, as implicitly defined by the FOC implied by (11) or equivalently where $dZ^i/dG^i = 0$. Since $\rho^i > 0$ and from (10b) we have $dZ^j/dG^i < 0$, the above expression evaluated at G^i_A given any feasible G^j is necessarily negative.[71] ||

Proof of Proposition 4. Lemma 3 establishes each country's best-response function shifts inward under trade relative to its positioning under autarky. We also know, from (A.5) and the conflict technology (5), that at most one country's best-response function can be negatively related to the rival's arming in the neighborhood of the autarkic equilibrium. Thus, at most one country could choose to arm by more under trade than under autarky.

[69] Note that successive transfers of secure labor and capital resources from country 2 to country 1 within $\mathcal{S}_0$ eventually imply $\sigma - \gamma_2/\zeta^2 < 0$ and thus drive arming by both countries under free trade (effectively) to zero. More generally, uneven distributions of secure resources in $\mathcal{S}_0$ imply that the two countries produce different quantities of Z^i, and such differences cause their FOC's under trade (13) to differ in equilibrium, such that $G^1_T \neq G^2_T$ even though $G^1_A = G^2_A$.

[70] As one can verify, transfers of secure capital this time from country 1 to country 2, again starting from any distribution of secure resources in $\mathcal{S}_0$, generate similar effects.

[71] Equations (B.17) and (B.18) in Supplementary Appendix B.2 show how the parameter ρ^i simplifies in the cases of (i) non-tariff trade costs only and (ii) tariffs chosen non-cooperative and simultaneously with security policies.

Clearly, when neither country arms by more (i.e., $G_T^i < G_A^i$ for $i = 1, 2$), aggregate arming is lower under trade than under autarky. To establish that aggregate arming is *always* lower under (possibly costly) trade than under autarky, then, we need only consider the case where $G_T^i < G_A^i$ for one country $i \in \{1, 2\}$, but $G_T^j > G_A^j$ for the other country $j \neq i$. This ranking necessarily implies $\partial B_A^i/\partial G^j > 0$ for all $G^j \leq G_A^j$ while $\partial B_A^j/\partial G^i < 0$ for G^i close to G_A^i. Supplementary Appendix B.1 shows these relationships, which arise when one country i has a sufficiently larger amount of secure labor to land relative to its rival j such that $G_A^i > G_A^j$, imply that $\partial B_A^j/\partial G^i + 1 > 0$ holds for all (G^i, G^j).

Now, starting from any value of G_0^i on the downward sloping segment of country j's best-response function, fix the associated sum $G_0^i + B_A^j(G_0^i) = G^i + G^j$ and consider a marginal decrease in G^i. The result that $\partial B_A^j/\partial G^i + 1 > 0$ at $G^i = G_0^i$ implies the decrease in G^i below G_0^i necessarily raises B_A^j by less than the increase in G^j needed to keep $G^i + G^j$ constant at $G_0^i + B_A^j(G_0^i)$—i.e., $B_A^j(G^i) < G^j$ for $G^i = G_0^i - \epsilon$, where $\epsilon > 0$. It then follows that $B_A^j(G^i) < G_A^j$ for any $G^i < G_A^i$ and G^j such that $G^i + G^j = G_A^i + G_A^j$, which implies in turn that $G^i + B_A^j(G^i) < G^i + G^j = G_A^i + G_A^j$. But, since $B_T^j(G^i) < B_A^j(G^i)$, we must have $G^i + B_T^j(G^i) < G_A^i + G_A^j$. Finally, let $G^i = G_T^i$, which implies $G^i + B_T^j(G^i) = G_T^i + G_T^j < G_A^i + G_A^j$, thereby completing the proof. ||

Proof of Proposition 5. From(14), country i's ($\neq j = 1, 2$) FOC for an interior solution with $t^i = t^j = 1$ can be written as:

$$\frac{dV_T^i/dG^i}{V_T^i} = \left(1 - \frac{\gamma_j^i}{\Delta}\right)\frac{dZ^i/dG^i}{Z^i} + \left(\frac{\gamma_j^i}{\Delta}\right)\frac{dZ^j/dG^i}{Z^j} = 0, \tag{A.9}$$

where, as previously defined in (14), $\Delta = \varepsilon^i + \varepsilon^j - 1$ and $\sigma > 1$ implies $\Delta > 1 > \gamma_j^i$. Now suppose that $G^i = G^j$ and that the value of G^i uniquely satisfies (A.9). We ask how G^i and G^j compare when $\tau^i \neq \tau^j$. Since $L^i = L$ and $K^i = K$ for $i = 1, 2$, $G^i = G^j$ implies $Z^i = Z^j$, $dZ^i/dG^i = dZ^j/dG^j > 0$ and $dZ^j/dG^i = dZ^i/dG^j < 0$. Then, from (A.9),

$$\text{sign}\left\{\partial V_T^j/\partial G^j|_{G^j=G^i}\right\} = \text{sign}\{\gamma_j^i - \gamma_i^j\} \tag{A.10}$$

holds. This expression implies that, if for example $\gamma_j^i > \gamma_i^j$, then $\partial V_T^j/\partial G^j|_{G^j=G^i} > 0$ and thus (provided, of course, $\partial^2 V_T^j/\partial(G^j)^2 < 0$), $G_T^j > G_T^i$ holds.

To identify how differences in trade costs influence differences between γ_j^i and γ_i^j, observe

$$\gamma_j^i - \gamma_i^j = \frac{(p_T^j)^{\sigma-1} - (p_T^i)^{\sigma-1}}{(1 + (p_T^i)^{\sigma-1})(1 + (p_T^j)^{\sigma-1})},$$

which implies, given $\sigma > 1$, that $\gamma_j^i \gtreqless \gamma_i^j$ as $p_T^i \lesseqgtr p_T^j$. Using equation (B.13) from Supplementary Appendix B.2 (assuming $\widehat{Z}^i = \widehat{Z}^j = 0$ as well as $t^i = t^j = 1$) with the arbitrage condition that $p^i = t^i \tau^i q^i$, we can see how how trade costs influence equilibrium prices:

$$\widehat{p}_T^i = \left(1 - \frac{\varepsilon^i - 1}{\Delta}\right)\widehat{\tau}^i + \left(\frac{\varepsilon^j - 1}{\Delta}\right)\widehat{\tau}^j. \tag{A.11}$$

Given our maintained assumption that $\sigma > 1$, one can easily verify that $\varepsilon^i > 1$ and $1 - \frac{\varepsilon^i - 1}{\Delta}$ $(= \frac{\varepsilon^j}{\Delta})$ > 0. Thus, an exogenous increase in τ^i increases both p_T^i and p_T^j.

To complete the proof, we need to verify $\tau^i \lesseqqgtr \tau^j$ implies $p_T^i \lesseqqgtr p_T^j$. To that end, let us consider a set of trade costs for countries i and j, with $\tau^i < \tau^j$, that imply a certain price p_T^i. Observe from (A.11) that combinations of τ^i and τ^j leaving p_T^i unchanged satisfy

$$\left.\frac{\widehat{\tau}^j}{\widehat{\tau}^i}\right|_{\widehat{p}_T^i = 0} = -\frac{1 - \left(\varepsilon^i - 1\right)/\Delta}{\left(\varepsilon^j - 1\right)/\Delta} = -\frac{\varepsilon^j}{\varepsilon^j - 1} < -1. \tag{A.12}$$

This expression implies that, as τ^i rises, τ^j must fall by a larger proportion to ensure that p_T^i remains unchanged. Turning to p_T^j, equation (A.11) implies

$$\widehat{p}_T^j = \left(\frac{\varepsilon^i}{\Delta}\right)\widehat{\tau}^j + \left(\frac{\varepsilon^i - 1}{\Delta}\right)\widehat{\tau}^i.$$

We now ask how p_T^j changes as τ^i rises and τ^j falls from their respective initial values to maintain p_T^i at its initial value. Solving for $\hat{\tau}^j|_{\widehat{p}_T^i = 0}$ from (A.12) and substituting that into the expression immediately above, one can confirm

$$\left.\widehat{p}_T^j\right|_{\widehat{p}_T^i = 0} = \left(\frac{\varepsilon^i}{\Delta}\right)\widehat{\tau}^j|_{\widehat{p}^i = 0} + \left(\frac{\varepsilon^i - 1}{\Delta}\right)\widehat{\tau}^i = -\frac{1}{\varepsilon^j - 1}\widehat{\tau}^i.$$

Thus, as τ^i rises, p_T^j falls along the schedule of trade costs that ensures p_T^i remains constant. But, as τ^i rises and τ^j falls to satisfy (A.12), we eventually reach the point where $\tau^i = \tau^j$ that implies $p_T^i = p_T^j$. As such, the initial trade-cost values with $\tau^i < \tau^j$ must have implied $p_T^i < p_T^j$ initially. More generally, we have $\tau^i \lesseqqgtr \tau^j$ implies $p_T^i \lesseqqgtr p_T^j$ and thus $\gamma_j^i \gtreqqless \gamma_i^j$, thereby with (A.10) completing the proof. $\quad ||$

Proof of Proposition 6.

Part a. Let us focus on an interior equilibrium in security policies under free trade.[72] Given our assumption that the two contending countries are identical in all respects means that, when $G^i = G$, $Z^1 = Z^2$, so that $\nu^i = \frac{1}{2}$, $w^i = w$, $r^i = r$, and $c^i = c$ for $i = 1, 2$. Assuming

[72]Establishing clean analytical conditions that ensure uniqueness generally for CES preferences is difficult. However, in the case of Cobb-Douglas preferences, matters simplify nicely, and uniqueness does hold. Moreover, numerical analysis assuming $\sigma > 1$ does not indicate the existence of multiple equilibria.

$\sigma > 1$, the FOCs under trade (16) can be written as

$$\text{sign}\left\{ \frac{\partial V_T^i}{\partial G^i}\bigg|_{G^i=G} \right\} = \text{sign}\left\{ ArK_0 \phi_{G^i}^i|_{G^i=G} - \psi \right\},$$

for $i = 1, 2$, where $A = 1 + \frac{\gamma_2/2\sigma}{1-\gamma_2/2\sigma} > 1$, for $\sigma < \infty$. Recall that the FOC for country i's arming choice under autarky (11) requires $dZ^i/dG^i = 0$ or equivalently $rK_0\phi_{G^i}^i|_{G^i=G_A} - \psi = 0$. Since $A > 1$, the sign of the FOC under trade when evaluated at $G^i = G_A$ for $i = 1, 2$ is strictly positive, thereby completing the proof.

Part b: To prove this part, we focus on cases where the strength of comparative advantage (i.e., $\alpha^i = a_2^i > 1$ for $i = 1, 2$) is sufficiently small to imply that world market-clearing price is very close to, but less than, the autarky price, such that the gains from trade are very small. Since the increased security costs implied by part (a) are independent of α^i, they can easily dwarf the small gains from trade. See Supplementary Appendix B.1 for details. ||

Proof of Proposition 7. In view of space constraints and the fact that the structure of this proof is very similar to that of Proposition 5, we only sketch it out here. (Supplementary Appendix B.2.4 provides details.) Following our previous strategy, we start with country i's FOC for an interior solution for G^i ($i = 1, 2$) in the presence of non-tariff trade costs:

$$\frac{dV_T^i/dG^i}{V_T^i} = \left(1 - \frac{\nu^i\gamma_2^i}{\Delta}\right)\frac{dZ^i/dG^i}{Z^i} - \left(\frac{\nu^j\gamma_2^i}{\Delta}\right)\frac{dZ^j/dG^i}{Z^j} = 0,$$

where ν^i represents country i's import share of good 2 and $\Delta = \sigma + (\sigma - 1)[-\gamma_1^3 + \nu^i\gamma_1^i + \nu^j\gamma_1^j] > \sigma$ assuming $\sigma > 1$. Observe that, when trade costs exist, domestic prices of good j differ across countries, such that expenditure shares for good 2 (γ_2^i) and import shares (ν^i) also differ across countries i, but we nonetheless have $\Delta > \nu^i\gamma_2^i$. Accordingly, since $dZ^j/dG^i < 0$ by (10b), an interior solution requires $dZ^i/dG^i < 0$.

Recalling our assumption that the two adversaries are identical, suppose $G^i = G^j$, where G^i uniquely satisfies the FOC above. Then, $Z^i = Z^j$, $dZ^j/dG^j = dZ^i/dG^i$, and $dZ^j/dG^i = dZ^i/dG^j$. To identify the ranking of equilibrium arming across adversaries $i = 1, 2$ who face different trade costs with friendly country $i = 3$, we evaluate $\text{sign}\{dV_T^j/dG^j|_{G^j=G^i}\}$ for $\tau^j \neq \tau^i$. Assuming $\partial^2 V_T^j/\partial(G^j)^2 < 0$ holds, a positive (negative) sign implies $G_T^j > G_T^i$ ($G_T^j < G_T^i$). As established in the full proof, if country 3's trade costs with countries $i = 1, 2$ are identical (i.e., $\tau_i^3 = \tau_j^3$), then only differences in expenditures shares matter for this calculation. More specifically, $\text{sign}\{dV_T^j/dG^j|_{G^j=G^i}\} = \text{sign}\{\gamma_2^j - \gamma_2^i\} = \text{sign}\{\tau^i - \tau^j\}$ holds, thereby completing the proof. ||

B Supplementary Appendix for "Arming in the Global Economy: The Importance of Trade with Enemies and Friends."

This supplementary appendix has three main sections. The first section (B.1) provides a proof of Lemma A.1 and presents the more technical details of the proofs presented in Appendix A. It also contains an additional proposition regarding the effects of the elasticity of substitution on equilibrium arming in the case of complete symmetry and its proof. The second section (B.2) extends the analysis to include trade costs, in support of the analysis in Sections 4.2 and 5, and presents some details of the numerical analysis including additional results; this section also includes a proof of Proposition 7. The final section (B.3) provides additional details regarding the empirical analysis in Section 6.

B.1 Additional Technical Details

Proof of Lemma A.1. That the equilibrium wage-rental ratio ω^i is independent of the trade regime in place, for given gross endowments and guns, follows directly from the equilibrium condition in (7).

To establish parts (a)–(e) of the lemma, let us temporarily omit country superscripts. Recall that

$$Z(\omega, K_g, L_g, G) = \frac{wL_g + rK_g - \psi G}{c(w,r)} = \frac{\omega L_g + K_g - \psi(\omega,1)G}{c(\omega,1)}. \tag{B.1}$$

Now differentiate Z with respect to w, K_g, L_g, and G to obtain the following (after some algebra):[1]

$$\begin{aligned}
\frac{\partial Z}{\partial w} &\equiv Z_w = \left(\frac{rc_w}{c^2}\right)(L_g - \psi_w G)\left[\frac{c_r}{c_w} - \frac{K_g - \psi_r G}{L_g - \psi_w G}\right] && \text{(B.2a)}\\
\frac{\partial Z}{\partial K_g} &\equiv Z_K = r/c && \text{(B.2b)}\\
\frac{\partial Z}{\partial L_g} &\equiv Z_L = w/c && \text{(B.2c)}\\
\frac{\partial Z}{\partial G} &\equiv Z_G = -\psi/c. && \text{(B.2d)}
\end{aligned}$$

Taking derivatives of the above expressions with respect to w shows

$$\begin{aligned}
Z_{ww} &= -(c_{ww}Z + \psi_{ww}G)/c > 0 && \text{(B.3a)}\\
Z_{Kw} &= -rc_w/c^2 < 0 && \text{(B.3b)}\\
Z_{Lw} &= rc_r/c^2 > 0 && \text{(B.3c)}\\
Z_{Gw} &= -\frac{\psi_w c - c_w \psi}{c^2} = \frac{r(\psi_r c_w - \psi_w c_r)}{c^2} = \frac{r\psi_w c_w}{c^2}\left(\frac{\psi_r}{\psi_w} - \frac{c_r}{c_w}\right). && \text{(B.3d)}
\end{aligned}$$

[1] We abstract here from the dependence of K_g on guns by treating gross factor endowments as exogenous; however, we explicitly consider this dependence in our analysis of endogenous security policies.

Part (a). As can be seen from (B.1), to trace the role of ω it is sufficient to consider the impact of variations in w keeping r fixed in the background. The expression inside the last set of square brackets in (B.2a) coincides with the domestic market-clearing condition (7) that produces the solution for ω; therefore, $Z_w = 0$ evaluated at the value of ω (ω^e) or, equivalently, $Z_\omega(\omega^e, \cdot) = 0$. Thus, the first component of this part of the lemma follows from (B.2a). The second component follows from the sign of the expression for Z_{ww} in (B.3a), as implied by (8), (7), and the linear homogeneity of the unit costs functions ψ and c, as well as their concavity in factor prices. That Z^e is strictly quasi-convex in ω and minimized at the equilibrium value implies that Z^e is the envelope function of $Z(\omega, \cdot)$, which is useful for establishing the remaining parts of the lemma and for our subsequent characterization of equilibria in security policies.

Part (b). This part follows from the implicit function theorem applied to $Z^e_w = 0$, while using (B.3). The first two components conform to intuition that an increase in the relative supply of one factor (capital or labor) decreases its relative price in factor markets. The last component reveals that, if the technology for guns is labor intensive (i.e., $c_r/c_w > \psi_r/\psi_w$), then, at constant factor prices, production of an additional gun will increase the demand for labor relative to capital, thus forcing the wage-rental ratio to rise.

Parts (c)–(e). The proof of the first component of parts (c), (d) and (e) follows readily from (B.2b)–(B.2d) with $Z^e_\omega = 0$ and the fact that $dZ^e = Z^e_\omega d\omega^e + Z^e_K dK_g + Z^e_L dL_g + Z^e_G dG$. The strict concavity of Z^e in, say, K_g can be proven as follows. First, observe that $dZ^e/dK_g = Z^e_\omega(d\omega^e/dK_g) + Z^e_K$. Now differentiate this expression with respect to K_g to find

$$\frac{d^2 Z^e}{dK_g^2} = Z^e_{\omega\omega}\left(\frac{d\omega^e}{dK_g}\right)^2 + 2Z^e_{\omega K}\frac{d\omega^e}{dK_g} + Z^e_\omega \frac{d^2\omega^e}{dK_g^2} + Z^e_{KK}.$$

Since $Z^e_\omega = 0$, $Z^e_{KK} = 0$, and $d\omega^e/dK_g = -Z^e_{\omega K}/Z^e_{\omega\omega}$, the expression above simplifies as

$$\frac{d^2 Z^e}{dK_g^2} = -\frac{(Z^e_{\omega K})^2}{Z^e_{\omega\omega}} < 0,$$

giving us the desired result. The strict concavity of Z^e in L_g and in G in parts (d) and (e) respectively can be shown along the same lines. Parts (c) and (d) state that Z^e is increasing and strictly concave in the country's gross factor endowments. Part (e) establishes that, given K_g, Z^e is decreasing and strictly concave in G. ||

Proof of Proposition 1 continued. To derive the sufficient conditions for uniqueness of the equilibrium in security policies under autarky as stated in the proposition, we demonstrate that the expression for $H^i \equiv H^i_1/H^i_2$ shown in equation (A.8) of Appendix A is less than one. Because the numerator (H^i_1) and the denominator (H^i_2) are both positive, we can subtract the former from the latter to obtain

$$C^i = H_2^i - H_1^i = -\frac{\phi^i \phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LG} - \left(-\frac{\phi^i \phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LZ} \right).$$

If $C^i > 0$, then $H^i < 1$ holds. The properties of $\phi(G^i, G^j)$ imply

$$\begin{aligned} -\frac{\phi^i \phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}} &= \frac{1}{\phi^j}\left[2\phi^i - \frac{f(G^i)f''(G^i)}{f'(G^i)f'(G^i)}\right] > 0 \quad (\text{since } f'' \leq 0) \\ -\frac{\phi^i \phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}} &= \frac{\phi^i}{\phi^j} - 1. \end{aligned}$$

Using the above observations in C^i after rearranging terms gives

$$C^i = \frac{1}{\phi^j}\left[\phi^i - \frac{f(G^i)f''(G^i)}{f'(G^i)f'(G^i)}\right] + \left[1 - \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LZ}\right] + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LG}.$$

A sufficient (but hardly necessary) condition for $C^i > 0$ is that the expression inside the second set of square brackets is non-negative. The definition of λ^i in (A.6) and the FOC under autarky (11) together imply after tedious algebra

$$\begin{aligned} \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LZ} &= \frac{r^i \phi^i K_0 \lambda^i}{\psi^i G^i}\theta^i_{LZ} = \frac{\theta^i_{LG}\theta^i_{LZ}\left(r^i\phi^i K_0\right)}{\sigma^i_G \theta^i_{LG}\theta^i_{KG}\psi^i G^i + \sigma^i_Z \theta^i_{LZ}\theta^i_{KZ} c^i Z^i} \\ &= \frac{\theta^i_{LG}\theta^i_{LZ}\left(r^i\phi^i K_0\right)}{\Upsilon^i + \theta^i_{LG}\theta^i_{KG}\psi^i G^i + \theta^i_{LZ}\theta^i_{KZ} c^i Z^i}, \end{aligned} \tag{B.4}$$

where $\Upsilon^i \equiv (\sigma^i_G - 1)\theta^i_{LG}\theta^i_{KG}\psi^i G^i + (\sigma^i_Z - 1)\theta^i_{LZ}\theta^i_{KZ} c^i Z^i$. Solving for G^i and Z^i from (6a) and (6b) with the definitions of the cost shares gives:

$$\begin{aligned} \psi^i G^i &= \frac{\theta^i_{KZ} w^i L^i - \theta^i_{LZ} r^i \left(K^i + \phi^i K_0\right)}{\theta^i_{KZ} - \theta^i_{KG}} \\ c^i Z^i &= \frac{-\theta^i_{KG} w^i L^i + \theta^i_{LG} r^i \left(K^i + \phi^i K_0\right)}{\theta^i_{KZ} - \theta^i_{KG}}, \end{aligned}$$

which together imply

$$\theta^i_{LG}\theta^i_{KG}\psi^i G^i + \theta^i_{LZ}\theta^i_{KZ} c^i Z^i = \theta^i_{KG}\theta^i_{KZ} w^i L^i + \theta^i_{LG}\theta^i_{LZ} r^i \left(K^i + \phi^i K_0\right).$$

Substitution of the above in (B.4) shows

$$\frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i}\theta^i_{LZ} = \left[\frac{\theta^i_{LG}\theta^i_{LZ} r^i \left(K^i + \phi^i K_0\right)}{\Upsilon^i + \theta^i_{KG}\theta^i_{KZ} w^i L^i + \theta^i_{LG}\theta^i_{LZ} r^i \left(K^i + \phi^i K_0\right)}\right]\left(\frac{\phi^i K_0}{K^i + \phi^i K_0}\right),$$

which is less than one, implying $C^i > 0$, $H^i < 1$ and thus uniqueness of equilibrium under autarky, if Υ^i is not very negative. A sufficient condition is that σ^i_G and/or σ^i_Z are not too much smaller than one. $\quad ||$

Proof of Proposition 2 continued.
Strict quasi-concavity of $\widetilde{V}^i_T$ in G^i. To present this proof, we introduce more compact notation. In particular, define $F^i \equiv Z^i(K^i_Z, L^i_Z)$ and let $F^i_{G^i} \equiv dZ^i/dG^i$ shown in (10a) and $F^j_{G^i} \equiv dZ^j/dG^i$ shown in (10b), for $j \neq i = 1, 2$, indicate the effects of a change in G^i on those optimized values. Then, with equation (12), we can rewrite country i's FOC under trade, focusing on interior solutions, as follows:

$$\frac{\partial \widetilde{V}^i_T}{\partial G^i} = \widetilde{V}^i_T \left[\frac{F^i_{G^i}}{F^i} - \frac{\gamma_j}{\sigma} \left(\frac{F^i_{G^i}}{F^i} - \frac{F^j_{G^i}}{F^j} \right) \right] = 0, \tag{B.6}$$

where $F^i_{G^i}/F^i - F^j_{G^i}/F^j > 0$ which implies $F^i_{G^i}/F^i > 0$. Note from (4) that $\partial p^i_T/\partial G^i = (p^i_T/\sigma)(F^i_{G^i}/F^i - F^j_{G^i}/F^j)$ and recall $\partial \gamma^i_j/\partial p^i_j = -(1-\gamma^i_j)(\sigma - 1)(\gamma^i_j/p^i_j)$. Differentiation of the expression above evaluated at an interior solution yields:

$$\begin{aligned}
\left.\frac{\partial^2 \widetilde{V}^i_T}{(\partial G^i)^2}\right|_{G^i = \widetilde{B}^i_T} &= \widetilde{V}^i_T \left\{ \frac{(\sigma-1)(1-\gamma_j)}{\gamma_j} \left(F^i_{G^i}/F^i\right)^2 \right. \\
&\quad + \left(1 - \frac{\gamma_j}{\sigma}\right) \left[\left(F^i_{G^i G^i}/F^i\right) - \left(F^i_{G^i}/F^i\right)^2 \right] \\
&\quad \left. + \left(\frac{\gamma_j}{\sigma}\right) \left[\left(F^j_{G^i G^i}/F^j\right) - \left(F^j_{G^i}/F^j\right)^2 \right] \right\}.
\end{aligned} \tag{B.7}$$

To evaluate the sign of this expression, we apply the implicit function theorem to $Z^i_\omega(\omega^i, \cdot) = 0$, using equations (B.3) and (10) with $F^i \equiv Z^i$ and (as before) attributing any implied changes in ω^i to changes in w^i alone:

$$\begin{aligned}
\left. w^i_{G^i} \right|_{G^i = \widetilde{B}^i_T} &= -\frac{K_0 Z^i_{wK} \phi^i_{G^i} + Z^i_{wG^i}}{Z^i_{ww}} = -\frac{c^i_w F^i_{G^i} + \psi^i_w}{c^i_{ww} F^i + \psi^i_{ww} G^i} > 0 \\
\left. w^j_{G^i} \right|_{G^i = \widetilde{B}^i_T} &= -\frac{K_0 Z^j_{wK} \phi^i_{G^i}}{Z^j_{ww}} = -\frac{c^j_w F^j_{G^i}}{c^j_{ww} F^j + \psi^j_{ww} G^j} < 0.
\end{aligned}$$

With these expressions, one can verify the following:

$$\begin{aligned}
\left. F^i_{G^i G^i} \right|_{G^i = \widetilde{B}^i_T} &= \frac{1}{c^i} \left[r^i K_0 \phi^i_{G^i G^i} + \frac{\left(c^i_w F^i_{G^i} + \psi^i_w\right)^2}{c^i_{ww} F^i + \psi^i_{ww} G^i} \right] < 0 \\
\left. F^j_{G^i G^i} \right|_{G^i = \widetilde{B}^i_T} &= \frac{1}{c^j} \left[r^j K_0 \phi^j_{G^i G^i} + \frac{\left(c^j_w F^j_{G^i}\right)^2}{c^j_{ww} F^j + \psi^j_{ww} G^j} \right].
\end{aligned}$$

Then, substitute the above into (B.7) and invoke the FOC under free trade (B.6), using the fact that $\phi^i_{G^i} = -\phi^j_{G^i}$. After rearranging, we have

$$\left.\frac{\partial^2 \widetilde{V}^i_T}{(\partial G^i)^2}\right|_{G^i=\widetilde{B}^i_T} = \widetilde{V}^i_T \left\{ -\left[\frac{1-\gamma_j+(\sigma-1)\gamma_j}{\gamma_j}\right](F^i_{G^i}/F^i)^2 \right.$$
$$+\left(\frac{1-\gamma_j/\sigma}{c^iF^i}\right)\frac{\phi^i_{G^iG^i}\psi^i}{\phi^i_{G^i}}+\left(\frac{1-\gamma_j/\sigma}{c^iF^i}\right)\frac{(c^i_wF^i_{G^i}+\psi^i_w)^2}{c^i_{ww}F^i+\psi^i_{ww}G^i}$$
$$\left.+\left(\frac{\gamma_j/\sigma}{c^jF^j}\right)\frac{(c^j_wF^j_{G^i})^2}{c^j_{ww}F^j+\psi^j_{ww}G^j}\right\}<0. \tag{B.8}$$

The negative sign follows from our assumptions that $\sigma > \gamma_j$ (to ensure that the marginal cost of arming is strictly positive) and $\phi^i_{G^iG^i}, c^i_{ww}, \psi^i_{ww} < 0$.

Sufficiently strong comparative advantage. We now turn to the condition in the proposition that α^i is sufficiently large. As noted earlier, for any given G^j, country i's choice of guns (G^i) influences its TOT and thus its imported good price p^i_T (which equals the world relative price in the absence of trade costs). However, this price cannot rise above the analogous price in country i under autarky, $p^i_A = \alpha^i$; nor can it fall below the relative domestic price of that good that prevails in country j, $1/p^j_A = 1/\alpha^j$. Thus, we have $p^i_T \in \left[1/\alpha^j, \alpha^i\right]$. In the main text we refer to α^i as the strength of country i's comparative advantage in producing good i. The smaller are α^i and α^j, the smaller is the range of prices within which country i's domestic price for its imported good can lie. This limited range, in turn, can lead to the emergence of multiple peaks in each country's payoff function given the opponent's guns choice that might generate discontinuities in best-response functions and, thus, can imply the non-existence of a pure-strategy equilibrium.

Above, we analyzed the outcome in security policies (G^1_T, G^2_T) based on the countries' unconstrained best-response functions, $\widetilde{B}^i_T(G^j)$ $i = 1, 2$ that ignore these boundary conditions. The requirement for (G^1_T, G^2_T) to be an equilibrium point is that it belongs to both countries' constrained best-response functions, $B^i_T(G^j)$ for $i = 1, 2$ that take into account the boundaries on the feasible range for p^i_T. The potential problem is that depending on the value of α^i, $B^i_T(G^j)$ could consist of two segments in the neighborhood of G^j_T, one that lies on $B^i_A(G^j)$ and another that lies on $\widetilde{B}^i_T(G^j)$, as illustrated in Fig. B.1. Thus, whether (G^1_T, G^2_T) qualifies as a pure-strategy equilibrium or not hinges on the location of the discontinuity relative to G^j_T. We now show that location depends on the strength of the two countries' comparative advantage, α^i for $i = 1, 2$.

To proceed, observe that, from (9) for $J = A$ and the fact that $p^i_A = \alpha^i$, we can write $V^i_A(G^i, G^j) = V^i_A(G^i, G^j; \alpha^i)$, where $\partial V^i_A/\partial\alpha^i < 0$; and, similarly from (9) for $J = T$, we can write $\widetilde{V}^i_T\left(G^i, G^j\right) = V^i_T\left(p^i_T\left(G^i, G^j\right), G^i, G^j\right)$, which gives us country i's payoff under trade when it accounts for the TOT effect, without imposing the constraint that

$p_T^i \leq \alpha^i$. Note especially that the strength of comparative advantage α^i has no direct effect on unconstrained payoffs under trade, $\partial \widetilde{V}_T^i / \partial \alpha^i = 0$.

Now consider a feasible value of G^j, and the following two values of α^i for country i associated with that level of arming by the rival:

(i) $\underline{\alpha}^i \equiv p_T^i(\widetilde{B}_T^i\left(G^j\right), G^j)$ denotes the value of α^i for which, given G^j, the TOT effect of country i's arming just vanishes, such that

$$V_A^i(\widetilde{B}_T^i(G^j), G^j; \underline{\alpha}^i) = \widetilde{V}_T^i(\widetilde{B}_T^i\left(G^j\right), G^j);$$

(ii) $\overline{\alpha}^i \equiv p_T^i(B_A^i(G^j), G^j)$ denotes the value of α^i that, given G^j, makes country i's payoff under autarky just equal to its payoff under trade if it were to operate along B_A^i, ignoring the TOT effect of its arming decision:

$$\widetilde{V}_T^i\left(B_A^i\left(G^j\right), G^j\right) = V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right).$$

Observe that $\partial p_T^i / \partial G^i > 0$ for any $G^i < B_A^i(G^j)$. In addition, comparing the FOC under autarky (11) with the FOC under trade (13) shows that $\widetilde{B}_T^i\left(G^j\right) < B_A^i(G^j)$ for any feasible G^j. Thus, we have $\underline{\alpha}^i < \overline{\alpha}^i$. Furthermore, since $\partial V_A^i / \partial \alpha^i < 0$, $V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right)$ holds. Moreover, we have

$$V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right) > V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right).$$

The definition of $\underline{\alpha}^i$ and the fact that $V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > V_A^i(\widetilde{B}_T^i(G^j), G^j; \underline{\alpha}^i)$ give the first inequality. Intuitively, when $\alpha^i = \underline{\alpha}^i \equiv p_T^i(\widetilde{B}_T^i\left(G^j\right), G^j)$, prices are fixed, and country i enjoys no gains from trade; thus, country i can obtain a larger payoff under autarky by expanding G^i all the way to its its best-response level, $B_A^i(G^j)$. The validity of the second inequality follows from the definition of $\overline{\alpha}^i$ and the fact that $\widetilde{V}_T^i(\widetilde{B}_T^i\left(G^j\right), G^j) > \widetilde{V}_T^i\left(B_A^i\left(G^j\right), G^j\right)$. When $\alpha^i = \overline{\alpha}^i \equiv p_T^i(B_A^i(G^j), G^j)$, country i can improve its payoff by operating on its best-response function under trade, $\widetilde{B}_T^i\left(G^j\right)$, which takes into account the TOT effect of arming. Since V_A^i is continuously decreasing in α^i, there exists a value of α^i, $\alpha_0^i \in (\underline{\alpha}^i, \overline{\alpha}^i)$, that solves

$$V_A^i\left(B_A^i\left(G^j\right), G^j; \alpha_0^i\right) = \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right).$$

Note that this value of α_0^i generally depends on G^j. Furthermore, there exist combinations of G^i and G^j such that $p_T^i(G^i, G^j) = \alpha_0^i$.

Let $\Lambda^i\left(G^j, \alpha^i\right)$ denote the difference in payoffs when country i operates on $\widetilde{B}_T^i(G^j)$ and when it operates on $B_A^i(G^j)$ given α^i and G^j, for $i \neq j = 1, 2$:

$$\Lambda^i\left(G^j, \alpha^i\right) \equiv \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right) - V_A^i\left(B_A^i\left(G^j\right), G^j; \alpha^i\right).$$

This function helps pin down possible discontinuities in player i's constrained best-response function under free trade, $B_T^i\left(G^j;\alpha^i\right)$, and clarifies their dependence on α^i. For any given G^j and α^i, we have

$$B_T^i(G^j;\alpha^i)=\begin{cases}\widetilde{B}_T^i(G^j) & \text{if} \quad \Lambda^i\left(G^j,\alpha^i\right)\geq 0\\ B_A^i(G^j) & \text{if} \quad \Lambda^i\left(G^j,\alpha^i\right)\leq 0.\end{cases}$$

Since $\partial V_A^i/\partial\alpha^i<0$ and $\widetilde{V}_T^i$ is independent of α^i, $\partial\Lambda^i/\partial\alpha^i\equiv\Lambda_{\alpha^i}^i>0$. Of course, the sign of $\Lambda^i\left(G^j,\alpha^i\right)$ also depends on G^j. With the envelope theorem, the negative externality that the rival's arming confers on country i implies $\partial\widetilde{V}_T^i/\partial G^j<0$ and $\partial V_A^i/\partial G^j<0$ for feasible G^j. Thus, unless we impose additional structure on the model, we cannot sign $\partial\Lambda^i/\partial G^j\equiv\Lambda_{G^j}^i$. $\Lambda^i\left(G^j,\alpha^i\right)$ could change signs (positive to negative or vice versa) or not change at all as G^j changes. This last possibility means that the equality $\Lambda^i\left(G^j,\alpha_0^i\right)=0$ need not imply $B_T^i(\cdot)$ is discontinuous at G^j given $\alpha^i=\alpha_0^i$.[2]

Nevertheless, assuming $\Lambda_{G^j}^i\neq 0$ so that $\Lambda^i\left(G^j,\alpha_0^i\right)=0$ does imply a discontinuity in $B_T^i(G^j)$, we can be more precise about how values of α^i for $i=1,2$ matter for the existence of a pure-strategy equilibrium under trade. Suppose $G^j=G_T^j$, and find the value of α^i (denoted by α_{0T}^i) that implies $\Lambda^i(G_T^j,\alpha_{0T}^i)=0$. Since $\Lambda_{\alpha^i}^i>0$, we know that $\Lambda^i(G_T^j,\alpha^i)>0$ for all $\alpha^i>\alpha_{0T}^i$ at $G^j=G_T^j$. Furthermore, for $\alpha^i=\alpha_{0T}^i$, a marginal increase in G^j above G_T^j implies $\Lambda^i(G^j,\alpha^i)\gtreqless 0$ as $\Lambda_{G^j}^i\gtreqless 0$. When α^i rises marginally above α_{0T}^i, the value of G^j that restores the equality $\Lambda^i(G^j,\alpha^i)=0$ will change according to $dG^j/d\alpha^i|_{\Lambda^i=0}=-\Lambda_{\alpha^i}^i/\Lambda_{G^j}^i$: $dG^j/d\alpha^i|_{\Lambda^i=0}\lesseqgtr 0$ iff $\Lambda_{G^j}^i\gtreqless 0$.

To see the implications of these observations, suppose that $\Lambda_{G^j}^i>0$ in the neighborhood of G_T^j, which implies $\Lambda^i(G^j,\alpha_{0T}^i)>0$ for G^j marginally above G_T^j, as illustrated in Fig. B.1.[3] In this case, a small increase in α^i above α_{0T}^i implies not only $\Lambda(\alpha^i,G_T^j)>0$ at G_T^j, but also the value of G^j that restores the equality $\Lambda(\alpha^i,G^j)=0$ falls. Thus, $B_T^i(G^j)$ shifts back from $B_A^i(G^j)$ to $\widetilde{B}_T^i(G^j)$ for values of G^j just below G_T^j given values of α^i just above α_{0T}^i, such that $B_T^i(G^j)=\widetilde{B}_T^i(G^j)$ over a larger range of G^j in the neighborhood of G_T^j. Alternatively, when $\Lambda_{G^j}^i<0$ so that $\Lambda^i(G^j,\alpha_{0T}^i)>0$ for G^j just below G_T^i, the marginal increase in α^i above α_{0T}^i implies that the point of discontinuity is above G_T^j. Then, for values of G^j just above G_T^j, $B_T^i(G^j)$ shifts back from $B_A^i(G^j)$ to $\widetilde{B}_T^i(G^j)$. As such, the

[2] For a given combination of $\alpha^i=\alpha_0^i$ and G^j that implies $\Lambda^i\left(G^j,\alpha^i\right)=0$, it is possible that $\Lambda_{G^j}^i=0$ for values of G^j in the neighborhood of that point of indifference, in which case country i would remain indifferent between trade and autarky in that neighborhood. Furthermore, it is possible that country i favors trade for all other possible values in G^j, or alternatively that it favors autarky for all other possible values of G^j. Although we cannot rule out such possibilities, we view them as highly unlikely, and focus on cases where $\Lambda_{G^j}^i\neq 0$.

[3] Thus, for G^j above the point of discontinuity (G_T^j), $B_T^i(G^j)=\widetilde{B}_T^i(G^j)$; and for $G^j<G_T^j$, $B_T^i(G^j)=B_A^i(G^j)$. In the case that $\Lambda_{G^i}^i<0$, $B_T^i(G^j)=\widetilde{B}_T^i(G^j)$ for G^j below the discontinuity and $B_T^i(G^j)=B_A^i(G^j)$ for points above it.

marginal increase in α^i implies once again that the range of G^js in the neighborhood of G^j_T for which $B^i_T(G^j) = \widetilde{B}^i_T\left(G^j\right)$ expands. Thus, the larger is the degree of comparative advantage α^i for country i relative to α^i_{0T}, the more likely it is that $\widetilde{B}^i_T(G^j_T) \in B^i_T(G^j_T)$. Since this is true for both i, (G^1_T, G^2_T) will be an equilibrium point under trade if $\alpha^i > \alpha^i_{0T}$ for both i.[4] $\quad ||$

Proposition B.1 (*Equilibrium arming and the elasticity of substitution under complete symmetry.*) *Suppose the conditions of complete symmetry are satisfied and that labor and capital are sufficiently substitutable in the production of arms and/or the intermediate good. Furthermore, assume that each country's comparative advantage* (α^i) *is sufficiently strong. Then, an increase in the elasticity of substitution in consumption* $\sigma > 1$ *induces greater arming under trade* (*i.e.,* $dG_T/d\sigma > 0$)*. As* σ *approaches* ∞*, equilibrium arming under trade approaches equilibrium arming under autarky* (*i.e.,* $\lim_{\sigma\to\infty} G_T = G_A$)*.*

Proof: The implications of complete symmetry for any given trade regime as detailed in the proof of Proposition 3 (in Appendix A) together with the assumption that $\sigma > 1$ allow us to write the FOC under trade (13) evaluated at $G^i = G_T > 0$ as

$$\frac{1}{m^i_T}\frac{\partial V^i_T}{\partial G^i} = \frac{1}{c}\left[\left(1 - \frac{1}{\sigma}\right) rK_0\phi^i_{G^i}\big|_{G^i=G_T} - \left(1 - \frac{1}{2\sigma}\right)\psi\right] = 0,$$

where we again introduce more compact notation $m^i_T \equiv m(p^i_T)$. Factoring out $1 - 1/2\sigma$ from the RHS allows us to rewrite this condition as

$$\Omega^i(G_T, \sigma) = ArK_0\phi^i_{G^i}\big|_{G^i=G_T} - \psi = 0,$$

where now $A = (\sigma - 1)/(\sigma - \frac{1}{2}) \in (0, 1)$. A comparison of the expression above with the analogous condition under autarky in (11) after imposing the conditions of complete symmetry $G^i_A = G_A$ (i.e., $rK_0\phi^i_{G^i}\big|_{G^i=G_A} - \psi = 0$) reveals the conditions that guarantee the existence of a unique, symmetric equilibrium in security policies under autarky ($G_A > 0$ for all $\sigma > 0$) also imply $\Omega^i_G < 0$; thus, provided comparative advantage is sufficiently strong, these conditions guarantee the existence of a unique, symmetric equilibrium under free trade: $G_T > 0$ for all $\sigma > 1$.[5] What's more, since $A < 1$ for $\sigma \in (1, \infty)$, we have $G_T \in (0, G_A)$. Our claim in the proposition that G_T is increasing in σ (for $\sigma > 1$) follows immediately since A is independent of the common level of guns and is increasing in σ. To verify the limit part of the proof, observe that $\lim_{\sigma\to\infty} A = 1$. $\quad ||$

Proof of Proposition 4 continued. Here, we demonstrate that $\partial B^j_A/\partial G^i + 1 > 0$ holds

[4] If $\alpha^i < \alpha^i_{0T}$, $G^i_T \neq \widetilde{G}^i_T$ and the equilibrium under free trade will involve mixed strategies and/or it will coincide with autarky.

[5] This finding also holds in the presence of symmetric trade costs.

for all (G^i, G^j) that satisfy $\partial B_A^j/\partial G^i < 0$ and $\partial B_A^i/\partial G^j > 0$. To this end, first recall from the proof of Proposition 1 that $\partial B_A^i/\partial G^j = (-\phi_{G^j}^i/\phi_{G^i}^i)H^i$, where H^i is given by (A.8). Economizing on notation, let $\Phi \equiv \frac{\phi^j \lambda}{\phi_{G^j}^j G^j} > 0$, $x \equiv \theta_{LZ}^j > 0$ and $y \equiv \theta_{LG}^j > 0$. Using these definitions and the specification of ϕ^i in (5), we have

$$\frac{\partial B_A^j}{\partial G^i} = \left(-\frac{\phi_{G^i}^j}{\phi_{G^j}^j}\right)\left(\frac{-\frac{\phi^j \phi_{G^j G^i}^j}{\phi_{G^i}^j \phi_{G^j}^j} + \Phi x}{-\frac{\phi^j \phi_{G^j G^j}^j}{\phi_{G^j}^j \phi_{G^j}^j} + \Phi y}\right) = \left[\frac{f(G^j)f'(G^i)}{f(G^i)f'(G^j)}\right]\left(\frac{\frac{f(G^j)}{f(G^i)} + \Phi x - 1}{2\frac{f(G^j)}{f(G^i)} - \frac{f(G^j)f''(G^j)}{(f'(G^j))^2 \phi^i} + \Phi y}\right),$$

where $f(\cdot) > 0$, $f'(\cdot)$ and $f''(\cdot) \leq 0$. Focusing on the second expression on the RHS, observe that the denominator of the second (multiplicative) term is positive. Thus, the sign of $\partial B_A^j/\partial G^i + 1$ coincides with the sign of the following

$$\begin{aligned} C &= \left[\frac{f(G^j)\, f'(G^i)}{f(G^i)\, f'(G^j)}\right]\left[\frac{f(G^j)}{f(G^i)} + \Phi x - 1\right] + 2\frac{f(G^j)}{f(G^i)} - \frac{f(G^j)\, f''(G^j)}{(f'(G^j))^2 \phi^i} + \Phi y \\ &= \frac{f(G^j)}{f(G^i)}\left[2 - \frac{f'(G^i)}{f'(G^j)}\right] + \left[\frac{f(G^j)\, f'(G^i)}{f(G^i)\, f'(G^j)}\right]\left[\frac{f(G^j)}{f(G^i)} + \Phi x\right] + \Phi y - \frac{f(G^j)\, f''(G^j)}{(f'(G^j))^2 \phi^i}. \end{aligned}$$

The last two terms of the second line are non-negative. Furthermore, the assumptions that $f'' < 0$ and $G^i > G^j$ together imply $f'(G^i)/f'(G^j) \leq 1$. Thus, $C > 0$, thereby completing the proof. ||

Proof of Proposition 6 continued. The proof of part (b) of the proposition builds on the feature of the model that, as in the analysis with only two countries, the world price is bounded by the two autarky prices: $p_T \in [1/p_A^3, p_A]$. To keep matters simple but without loss of generality, we suppose that $a_1^1 = a_1^2 = a_2^3 = 1$ and $a_2^1 = a_2^2 = a_1^3 = \alpha > 1$, so that countries 1 and 2 have an identical comparative advantage in the production of good 1, whereas country 3 has a comparative advantage in the production of good 2; and, furthermore, comparative advantage is symmetric across countries 1 and 2 on the one hand and country 3 on the other, with $\alpha > 1$ indicating the strength of that advantage. Given these simplifications, $p_A^i = \alpha$ for $i = 1, 2, 3$. To simplify a little more, suppose in addition that preferences are Cobb-Douglas ($\sigma = 1$).[6] Then, the condition of balanced trade implies

$$p_T = \begin{cases} 1/\alpha & \text{if} \quad \pi \leq 1/\alpha \\ \pi & \text{if} \quad 1/\alpha < \pi < \alpha \\ \alpha & \text{if} \quad \alpha \leq \pi, \end{cases}$$

where $\pi \equiv \frac{\gamma_2}{\gamma_1}\left[\frac{Z^1 + Z^2}{Z^3}\right]$. Since the expenditure shares ($\gamma_1 = 1 - \gamma_2$ for all three countries)

[6]The analysis does not change substantively if we allow $\sigma > 1$. In any case, note that we are not restricting preferences here to be symmetrically defined across the two goods.

are constant, the expression above shows that p_T remains fixed for combinations of Z^1 and Z^2 that imply (given Z^3) a constant sum $Z^1 + Z^2$.

We assume further (for clarity) that guns are produced with labor only (so that $\psi^i = w^i$), which implies that $Z^i = Z(L-G^i, K+\phi^i K_0)$, where $Z(\cdot)$ now represents a standard constant returns to scale production function that gives the output of the intermediate good Z^i.[7] Then, the relevant upper bound on the world price can be written as

$$\sum\nolimits_{i=1,2} Z^i = \sum\nolimits_{i=1,2} Z(L - G^i, K + \phi^i K_0) = \frac{\gamma_1}{\gamma_2}\alpha Z^3 \equiv \overline{Z},$$

which defines combinations of guns such that $p_T = \alpha$.[8] This constraint is depicted in Fig. B.2, which also shows country 1's best-response function under autarky $B_A^1(G^2)$ and its best-response function under free trade that ignores the constraint on p_T, $\widetilde{B}_T^1(G^2)$.[9] Combinations of guns above the $p_T = \alpha$ constraint imply $p_T < \alpha$, whereas combinations on and below the constraint imply $p_T = \alpha$ (or equivalently $\sum_{i=1,2} Z^i \geq \overline{Z}$). Starting at points where the constraint binds, changes in guns by either country would cause p_T to fall only if the new point is beyond the $p_T = \alpha$ constraint.

The shape of this constraint, as shown in the figure, has three key properties:

(i) The constraint has a slope of -1 where it intersects the 45° line, as can be confirmed by evaluating $dG^2/dG^1|_{p_T=\alpha}$ at $G^2 = G^1$.

(ii) It is not possible for both G^1 and G^2 to increase as we move rightward along the constraint away from the 45° line towards $\widetilde{B}_T^1(G^2)$.[10]

(iii) The properties above in turn imply that the $p_T = \alpha$ constraint must intersect (or approach) $\widetilde{B}_T^1(G^2)$ at some point C' below and to the right of its intersection with the 45° line.[11]

Larger values of α imply that the $p_T = \alpha$ constraint lies closer to the origin, without

[7] The assumption that guns are produced with labor alone is not crucial, but simplifies the analysis.

[8] A similar constraint can be written for the lower bound of $p_T = 1/\alpha$: $\sum_{i=1,2} Z^i = Z^3\gamma_1/(\gamma_2\alpha) \equiv \underline{Z}$, with $\underline{Z} < \overline{Z}$. However, it is the upper bound that is relevant here.

[9] That $B_A^1(G^2) < \widetilde{B}_T^1(G^2)$ for given G^2 reflects the effect of trade to augment a country's incentive to arm against another country when the two compete in the same export market, as emphasized in the text.

[10] This property follows from the effects of changes in arming on $\sum_{i=1,2} Z^i$. Starting at the point where the $p_T = \alpha$ constraint intersects $B_A^1(G^2)$ in Fig. B.2, let G^1 rise while keeping G^2 fixed. The FOC under autarky (11) implies $\partial V_A^1/\partial G^1 < 0$ for $G^1 > B_A^1(G^2)$, so that $dZ^1/dG^1 < 0$; since $dZ^2/dG^1 < 0$ holds too, the increase in G^1 causes $\sum_{i=1,2} Z^i$ and thus p_T to fall, implying that the new combination of guns is above the $p_T = \alpha$ constraint. Repeated applications of this logic establish that G^1 and G^2 cannot both increase as we continue to move along the constraint approaching $\widetilde{B}_T^1(G^2)$). That is to say, the $p_T = \alpha$ constraint cannot be U-shaped to the right of the 45° line. However, it is possible that $dG^2/dG^1|_{p_T=\alpha} \to \infty$ somewhere along the constraint as we move further towards or beyond $\widetilde{B}_T^1(G^2)$—a possibility illustrated in Fig. B.2. There exist no combinations of G^1 and G^2 beyond this critical point, whether it is located to the left or right of $\widetilde{B}_T^1(G^2)$, where the price constraint binds.

[11] C' can be arbitrarily close to point 0, but that does not matter for our argument to follow.

affecting the positioning of $B^1_A(G^2)$ and $\widetilde{B}^1_T(G^2)$.[12]

Now consider the value of G^2 denoted by G^2_C such that $G^1_A = B^1_A(G^2_C)$ implies $\sum_{i=1,2} Z^i > \overline{Z}$ and, more importantly, such that $G^1_T = \widetilde{B}^1_T(G^2_C)$ implies $\sum_{i=1,2} Z^i = \overline{Z}$. For $G^2 = G^2_C$, a shift from autarky to free trade (shown in Fig. B.2 as a movement from point C to point C') implies $p_T = p_A = \alpha$. With no gains from trade given $G^2 = G^2_C$, country 1 prefers to stay on its best-response function under autarky that simply maximizes its output of Z^1. That is to say, $V^1_A(B^1_A(G^2_C), G^2_C) > \widetilde{V}^1_T(\widetilde{B}^1_T(G^2_C), G^2_C)$. Next consider a larger value of G^2 denoted by G^2_D such that $G^1_A = B^1_A(G^2_D)$ implies $\sum_{i=1,2} Z^i = \overline{Z}$ and furthermore $G^1_T = \widetilde{B}^1_T(G^2_D)$ implies $\sum_{i=1,2} Z^i < \overline{Z}$. In this case as shown in Fig. B.2, a shift from autarky to free trade implies $p_T < \alpha$ and induces country 1 to increase its arming in a move from $B_A(G^2_D)$ (point D) to $\widetilde{B}_T(G^2_D)$ (point D'), with $\widetilde{V}^1_T(\widetilde{B}^1_T(G^2_D), G^2_D) > V^1_A(B^1_A(G^2_D), G^2_D)$. By continuity, there exists a value of arming by country 2, denoted by $G^2_0 \in (G^2_C, G^2_D)$, such that $V^1_A(B^1_A(G^2_0), G^2_0) = \widetilde{V}^1_T(\widetilde{B}^1_T(G^2_0), G^2_0)$. This value of G^2_0 defines the discontinuity in country 1's best-response function under (free) trade, denoted by $B^1_T(G^2)$:

$$B^1_T(G^2) = \begin{cases} B^1_A(G^2) & \text{if } G^2 \le G^2_0; \\ \widetilde{B}^1_T(G^2) & \text{if } G^2 \ge G^2_0. \end{cases}$$

Since the location of the $p_T = \alpha$ constraint depends on α (moving closer to the origin as α increases), the location of the discontinuity depends on α as well.

We flesh out the possible welfare implications here, with the help of Fig. B.2. Point A on the 45° line where the best-response functions of the two (identical) countries under autarky would cross (so that $G^1_A = G^2_A = G_A$) represents the unique, symmetric equilibrium under autarky. Point T also on the 45° line shows where the unconstrained best-response functions under trade would cross so that $G^1_T = G^2_T = G_T$. Provided T lies above the $p_T = \alpha$ constraint, it represents an equilibrium under free trade. A movement along the 45° line from A to T implies no change in the distribution of K_0, but higher security costs. How these added security costs compare with the gains from trade depends on the location of the discontinuity of the constrained best-response function under trade, $B^1_T(G^2)$.

Suppose that the discontinuity occurs at $G^2_0 = G_A$, depicted as point A. By the definition of the discontinuity, the payoffs to country 1 under autarky and trade will be equal at this level of arming by country 2: $V^1_A(B^1_A(G_A), G_A) = \widetilde{V}^1_T(\widetilde{B}^1_T(G_A), G_A)$. We now show that a movement along $\widetilde{B}^1_T(G^2)$ from $(\widetilde{B}^1_T(G_A), G_A)$ in the direction of point T reduces country 1's payoff in the 3-country case. By the envelope theorem, since we are moving along country 1's best-response function under trade, we need only to consider the welfare effects of a

[12] The $p_T = \alpha$ constraint likewise moves closer to the origin when either Z^3 or $\gamma_1 = 1 - \gamma_2$ increases, but such changes could also affect the positioning of $\widetilde{B}^1_T(G^2)$. Focusing on how the positioning of the $p_T = \alpha$ constraint depends on α allows us to show most clearly how the price constraint relates to the location of the discontinuity of the best-response function under free trade as derived below.

change in G^2 as it influences country 1's optimal production of the intermediate input and p_T. Keeping in mind the effects of a change in G^2 on both countries' optimizing production, dZ^1/dG^2 and dZ^2/dG^2 shown in (10), we use (15) with $\Delta = \sigma = 1$ and rearrange to find:

$$\frac{1}{V_T^1}\frac{\partial V_T^1}{\partial G^2} = \left[1 - \nu^1\gamma_2\right]\frac{dZ^1/dG^2}{Z^1} - [\nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2}. \tag{B.9}$$

To evaluate the sign of (B.9), observe that, for points on $B_T^1\left(G^2\right)$ where $G^2 < B_T^2\left(G^1\right)$, the net marginal benefit of arming for country 2 is positive:

$$\frac{1}{V_T^2}\frac{\partial V_T^2}{\partial G^2} = [1 - \nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2} - [\nu^1\gamma_2]\frac{dZ^1/dG^2}{Z^1} > 0.$$

This inequality can be rewritten as

$$-[\nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2} < -\left[\frac{\nu^1\nu^2\left(\gamma_2\right)^2}{1 - \nu^2\gamma_2}\right]\frac{dZ^1/dG^2}{Z^1},$$

and then combined with (B.9) (using the fact that $\nu^1 + \nu^2 = 1$) to find

$$\frac{1}{V_T^1}\frac{\partial V_T^1}{\partial G^2} < \left[\frac{1 - \gamma_2}{1 - \nu^2\gamma_2}\right]\frac{dZ^1/dG^2}{Z^1}.$$

Since $\nu_2, \gamma_2 < 1$ and $dZ^1/dG^2 < 0$, the RHS of the expression above is negative, which in turn implies that $\partial V_T^1/\partial G^2 < 0$.[13] As such, $V_T(G_T, G_T) < V_A(G_A, G_A)$ when $G^2 = G_A$.

By continuity, there exist higher values of α (with the $p_T = \alpha$ constraint and the discontinuity in $B_F^1(G^2)$ moving towards the origin), such that the gains from trade continue to be less than the higher security costs under trade, implying that a shift from autarky to free trade is welfare reducing. Of course, increases in the strength of comparative advantage eventually imply sufficiently large gains from trade that swamp the higher security costs and thus render free-trade Pareto preferred to autarky.[14] ||

[13] A sufficient (but not necessary) condition for this result to hold more generally under CES preferences is that $\sigma \geq 1$.

[14] Note that for smaller α, the discontinuity in $B_T^1(G^2)$ moves between points A and T. In such cases, both points A and T represent pure-strategy equilibria, with autarky being Pareto preferred to free trade. For α sufficiently small to push the $p_T = \alpha$ constraint beyond point T, the only pure-strategy equilibrium is autarky.

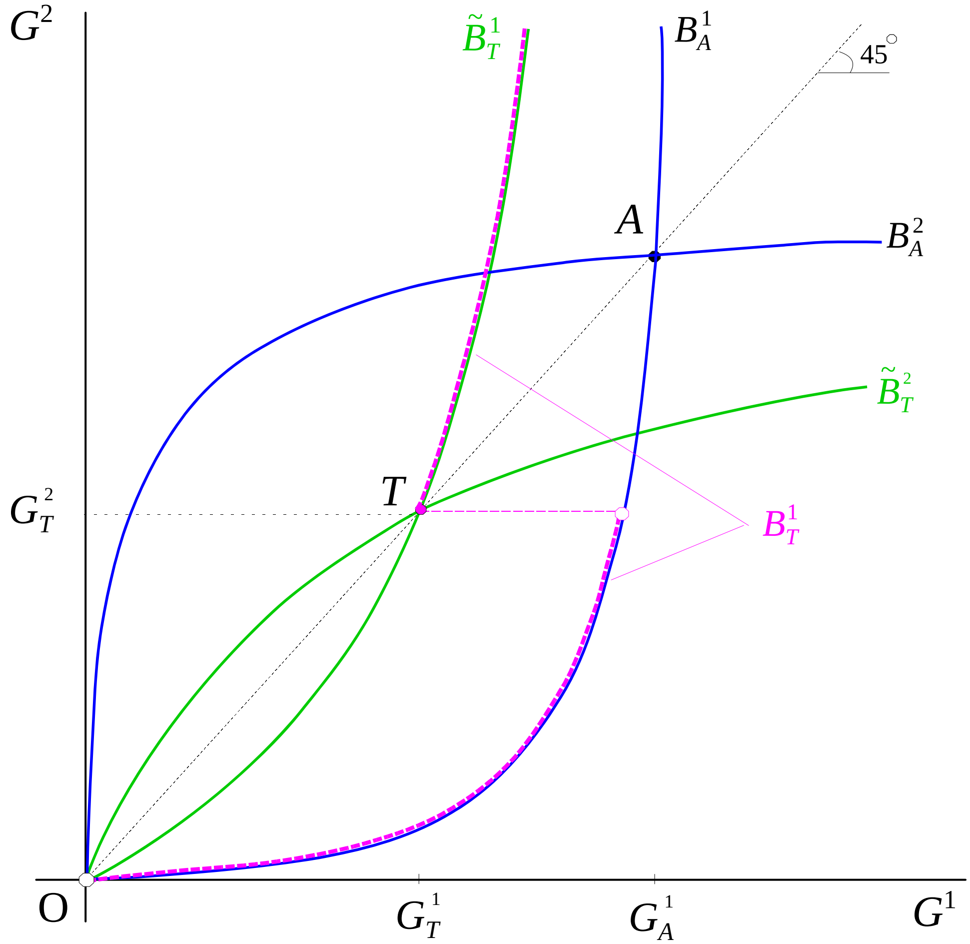

Figure B.1: Derivation of Best-Response Function under Trade and Nash Equilibria in Security Policies in the Two-Country Case

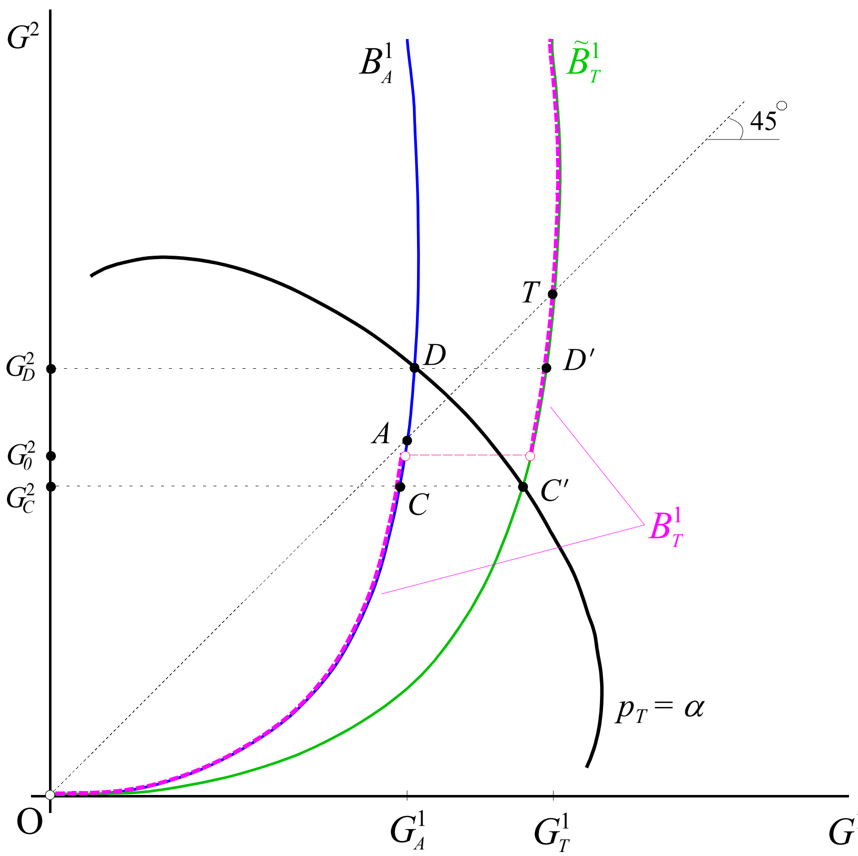

Figure B.2: Derivation of Best-Response Function under Free Trade and Comparison of Nash Equilibria in Security Policies in the Three-Country Case

B.2 Trade Costs

This part of the supplementary appendix focuses on the implications of trade costs in both the two-country setting (in the first three sections) and in the three-country setting.

B.2.1 Introducing Trade Costs between Adversarial Countries

Trade in world markets can be costly first due to the possible existence of geographic or physical trade barriers that take Samuelson's "iceberg" form. In particular, for each unit of a final good that country i imports (and consumes), country j must ship τ^i (≥ 1) units. Trade across national borders might be costly also due to the existence of import tariffs. Denote country i's (ad valorem) import tariff plus 1 by t^i (≥ 1).[15] Our analysis of trade must be modified in two principal ways to account for trade costs. First, we need to distinguish between domestic and world prices. Specifically, letting q_j denote the "world" price of good j, competitive pricing implies $q_j = p_j^j = c^j$ for $j = 1, 2$ and $q^i = q_j/q_i = c^j/c^i$. But, the presence of trade costs drives a wedge between this world relative price and the domestic relative price of that same good imported by country i, $p^i = p_j^i/p_i^i$. In particular, $p_T^i = t^i \tau^i q_T^i$, which implies $\widehat{p}_T^i = \widehat{t}^i + \widehat{\tau}^i + \widehat{q}_T^i$. Hereafter, where feasible, we omit the subscript "T" to avoid clutter.

Second, we must capture the presence of tariff revenues.[16] Assume that each country i specializes completely in producing the good i, so that its demand for good j is satisfied entirely through imports: $M^i = D_j^i$, where from (1) $D_j^i = \gamma_j^i Y^i / p_j^i$. Then,

$$Y^i = c^i Z^i + \tfrac{t^i - 1}{t^i} p_j^i D_j^i = c^i Z^i + \tfrac{t^i - 1}{t^i} \gamma_j^i Y^i \quad \Longrightarrow \quad Y^i = \frac{c^i Z^i}{\gamma_i^i + \gamma_j^i / t^i}, \quad i \neq j. \tag{B.10}$$

Substitution of the second expression for Y^i in (B.10) back into D_j^i gives

$$D_j^i = \frac{\gamma_j^i Z^i / p^i}{\gamma_i^i + \gamma_j^i / t^i}, \quad i \neq j = 1, 2. \tag{B.11}$$

The (absolute value of the) uncompensated (Marshallian) price elasticity of import demand ε^i and the (absolute value of the) compensated (Hicksian) price elasticity of import demand η^i can be computed respectively as follows:

$$\varepsilon^i \equiv -\frac{\partial M^i / \partial p^i}{M^i / p^i} = 1 + (\sigma - 1) \frac{t^i \gamma_i^i}{t^i \gamma_i^i + \gamma_j^i} \tag{B.12a}$$

$$\eta^i \equiv -\left.\frac{\partial M^i / \partial p^i}{M^i / p^i}\right|_{dU^i = 0} = \varepsilon^i - \frac{\gamma_j^i}{t^i \gamma_i^i + \gamma_j^i} = \sigma \left(\frac{t^i \gamma_i^i}{t^i \gamma_i^i + \gamma_j^i} \right) > 0, \tag{B.12b}$$

[15] We abstract from the possible existence of internal transportation costs and export taxes.

[16] We assume these revenues are distributed to consumers in a lump sum fashion.

for $i \neq j = 1, 2$. Note that $\operatorname{sign}\left\{\varepsilon^i - 1\right\} = \operatorname{sign}\left\{\sigma - 1\right\}$.

Given countries specialize completely in the production of their respective exported goods, world market clearing requires $\tau^i q_j D^i_j = \tau^j q_i D^j_i$. Using (B.11) and the pricing relations spelled out above, one can verify this condition implies:

$$q^i \left(\frac{\gamma^i_j Z^i}{t^i \gamma^i_i + \gamma^i_j}\right) = \frac{\gamma^j_i Z^j}{t^j \gamma^j_j + \gamma^j_i},\ i \neq j = 1, 2.$$

Thus, the world market-clearing price q^i is implicitly defined as a function of security policies and trade costs. In what follows, we assume this condition is satisfied for $p^i = \tau^i t^i q^i \in [1/\alpha^j, \alpha^i]$ for $i \neq j = 1, 2$, so as to abstract from the possible issues that can arise in relation to discontinuities in the best-response functions. More to the point, assuming a sufficiently large comparative advantage (α^i) for each country i, given trade costs, allows us to focus on the salient features of the trade equilibrium that involves strictly positive trade flows and thus is distinct from the equilibrium under autarky.

Totally differentiating this expression and rearranging terms allows us to trace out the effects of Z^i, Z^j, t^i, t^j, τ^i and τ^j on the market-clearing relative world price of country i's imported good, q^i:

$$\widehat{q}^i = \frac{1}{\Delta}\left[\widehat{Z}^i - \widehat{Z}^j - \eta^i \widehat{t}^i + \eta^j \widehat{t}^j - \left(\varepsilon^i - 1\right)\widehat{\tau}^i + \left(\varepsilon^j - 1\right)\widehat{\tau}^j\right], \tag{B.13}$$

where η^i and ε^i are shown in (B.12) and where we assume the Marshall-Lerner condition for stability, given by $\Delta \equiv \varepsilon^i + \varepsilon^j - 1 > 0$, is satisfied. As expected, an increase (decrease) in t^i (t^j) improves country i's TOT. Furthermore, if $\sigma > 1$, then a change in τ^i (τ^j) has a qualitatively similar effect on q^i. The effects of changes in Z^i and Z^j on q^i are as expected. If the percentage increase in Z^i (brought about by a change in, say, G^i) exceeds the corresponding change in Z^j, then i's TOT will deteriorate.

B.2.2 Payoff Functions and the Incentive To Arm in the Two-Country Setting

The payoff functions under costly trade are central to characterizing the countries' optimizing choices of guns production. Recall we can write $V^i = \mu^i Y^i$, where μ^i is the inverse of the price index, given by $P^i = p^i_i[1 + (p^i)^{1-\sigma}]^{1/(1-\sigma)}$. Then, using the expression for $Y^i = c^i Z^i = p^i_i Z^i$ shown in (B.10), we can rewrite the indirect utility function as

$$V^i = \frac{p^i_i Z^i / P^i}{\gamma^i_i + \gamma^i_j / t^i},\ \ i \neq j = 1, 2. \tag{B.14}$$

Total differentiation of (B.14), with a focus on percentage changes, yields

$$\widehat{V}^i = \widehat{Z}^i + \frac{\gamma^i_j}{t^i \gamma^i_i + \gamma^i_j}\widehat{t}^i - \frac{\gamma^i_j}{t^i}\left[(t^i - 1)\varepsilon^i + 1\right]\left(\widehat{t}^i + \widehat{\tau}^i + \widehat{q}^i\right). \tag{B.15}$$

Upon substituting the expression for $\widehat{q}_T^i$ (B.13) into (B.15) and simplifying, one can verify

$$\begin{aligned}\widehat{V}^i &= \left(1-\rho^i\right)\widehat{Z}^i + \rho^i \widehat{Z}^j - \rho^i\left[\varepsilon^j\widehat{\tau}^i + \left(\varepsilon^j - 1\right)\widehat{\tau}^j + \eta^j \widehat{t}^j\right] \\ &\quad + \left(\gamma_j^i/t^i\Delta\right)\eta^i\left[1-\left(t^i-1\right)\left(\varepsilon^j-1\right)\right]\widehat{t}^i, \end{aligned} \tag{B.16}$$

for $j \neq i = 1,2$, where $\rho^i \equiv (\gamma_j^i/t^i\Delta)[\left(t^i-1\right)\varepsilon^i + 1]$. The first two terms in the first line identify the effects of changes in Z^i and Z^j respectively. The next term in the first line identifies the effects of changes in non-tariff trade costs for countries i and j and tariffs imposed by country j. The term in the second line shows the effect of changes in tariffs imposed by the country i. Observe especially that this formulation allows for asymmetric trade costs across the two countries.

To explore how the presence of trade costs matters for arming incentives, we now consider the effect of an increase in country i's own guns G^i on its payoff. For example, if there are non-tariff barriers ($\tau^i > 1$) but no tariffs ($t^i = 1$ so that $\rho^i = \gamma_j^i/\Delta$), then (B.16) implies the following net marginal benefit from arming for country i:

$$\left.\frac{\partial V_T^i/\partial G^i}{V_T^i}\right|_{t^i=1} = \left(1-\frac{\gamma_j^i}{\Delta}\right)\frac{dZ^i/dG^i}{Z^i} + \left(\frac{\gamma_j^i}{\Delta}\right)\frac{dZ^j/dG^i}{Z^j}. \tag{B.17}$$

Observe that this expression is similar to what we have under free trade, shown in (12), with the only difference being the replacement of Δ for σ.[17] Alternatively, suppose each country simultaneously chooses its tariff and security policies to maximize its payoff. From (B.16), a necessary condition for i's tariff to be optimal (given $\sigma > 1$) is that $t^i - 1 = 1/(\varepsilon^j - 1)$ for $i \neq j = 1,2$ or, equivalently, $t_o^i = \varepsilon^j/(\varepsilon^j - 1)$, which is the standard optimal tariff formula. From the definition of $\Delta \equiv \varepsilon^i + \varepsilon^j - 1 > 0$, we have $\rho^i = \gamma_j^i/\varepsilon^j$; and, from (B.16), country i's net marginal benefit from arming in percentage terms becomes:

$$\left.\frac{\partial V_T^i/\partial G^i}{V_T^i}\right|_{t^i=t_o^i} = \left(1-\frac{\gamma_j^i}{\varepsilon^j}\right)\frac{dZ^i/dG^i}{Z^i} + \left(\frac{\gamma_j^i}{\varepsilon^j}\right)\frac{dZ^j/dG^i}{Z^j}. \tag{B.18}$$

Now recall, from (10b), that an increase in G^i given G^j implies less of the contested resource for country j—i.e., $dZ^j/dG^i < 0$ holds. Furthermore, while country i recognizes this effect in its choice of arms under trade, it ignores this effect under autarky. Instead, as established earlier, each country i under autarky chooses G^i effectively to maximize the income from its production of Z^i, which implies $dZ^i/dG^i = 0$. Accordingly, for each of the two cases above as well as more generally, country i's net marginal benefit from arming under trade evaluated at G_A^i given G^j is negative.[18] Hence, country i's incentive to arm (given G^j) is

[17] As one can easily verify, $\Delta = \sigma$ when $\tau^i = t^i = 1$.

[18] It is also possible to show, in a setting without tariffs, a decrease in either country i's non-tariff trade barriers (i.e., where initially $\tau^i > 1$) reduces each country's net marginal benefit from arming.

smaller under trade, whether costly or not. The only restriction we make here is that, given trade costs, the strength of comparative advantage is sufficiently large to allow for strictly positive trade flows.

B.2.3 Numerical Results for the Two-Country Setting

As was the case under free trade, a shift from an equilibrium under autarky to one with trade could induce one adversary to increase its arming. Because the equilibrium under (costly) trade is intractable analytically, we establish this point with the help of numerical methods. In addition, we extend the analysis to explore the dependence of arming on iceberg-type trade costs and tariffs. Finally we compare the various equilibria to the ones that would arise if countries chose their tariffs and security policies optimally.

To identify the effects of changes in trade costs on equilibrium arming choices and payoffs, let $V^i_{G^i} = 0$ $(i = 1, 2)$ be the FOC condition associated with country i's arming under trade. Now let s denote a non-tariff or policy related trade cost. We then ask what is the impact of an increase in s on the two countries' arming decisions. It is straightforward to show

$$\frac{dG^i_T}{ds} = \frac{1}{D}\left[\overset{(?)}{V^i_{G^iG^j}}\overset{(?)}{V^j_{G^js}} - \overset{(-)}{V^j_{G^jG^j}}\overset{(?)}{V^i_{G^is}}\right] \tag{B.19a}$$

$$\frac{dG^j_T}{ds} = \frac{1}{D}\left[-\overset{(-)}{V^i_{G^iG^i}}\overset{(?)}{V^j_{G^js}} + \overset{(?)}{V^j_{G^jG^i}}\overset{(?)}{V^i_{G^is}}\right], \tag{B.19b}$$

where $D \equiv V^i_{G^iG^i}V^j_{G^jG^j} - V^i_{G^iG^j}V^j_{G^jG^i}$. To ensure the equilibrium in arming is stable, we assume $D > 0$. We also assume $V^i_{G^iG^i} < 0$ and $V^j_{G^jG^j} < 0$, so that the second-order condition for each country's optimizing choice of guns is satisfied. Clearly, the impact of a change in s on arming depends on (i) the sign of its direct effect on both countries' marginal payoffs due to arming and, (ii) on whether their security policies are strategic substitutes or strategic complements.

Our numerical simulations are based on a particular parameterization of the model that assumes guns are produced by both countries with labor only and on a one-to-one basis, implying $Z^i = \left(K^i + \phi^i K_0\right)^{1-\theta}\left(L^i - G^i\right)^{\theta}$, where $\theta \in (0, 1)$. When the distribution of secure resources is sufficiently even to imply that both countries' security policies exhibit strategic complementarity in the neighborhood of the autarkic equilibrium, the analysis yields predictable results. Thus, while we consider such cases below, we focus largely on those cases where the distribution is extremely uneven, using figures to illustrate. The figures assume $\theta = 0.2$, $\alpha = \infty$, $K^1 = L^1 = 1.95$ and $K^2 = L^2 = 0.05$, with $K_0 = 1$.[19]

[19] These assumptions imply that the two countries initially hold an identical ratio of capital to labor. However, what matters in determining whether one country's best-response function in the neighborhood of the autarky equilibrium depends positively or negatively on its rival's arming is the similarity of their ratios

Non-tariff trade costs. Let us consider $s = \tau^i$, and assume neither country imposes tariffs: $t^i = t^j = 1$. Then, country i's FOC with respect to arming at an interior solution requires $\frac{\partial V^i/\partial G^i}{V^i}|_{t^i=1} = 0$. It is straightforward to show, using (B.17), that $V^i_{G^i\tau^i} > 0$ and $V^j_{G^j\tau^i} > 0$.[20] Accordingly, the following ideas can also be established with the help of numerical analysis.

(a) If the countries' secure factor endowments are sufficiently similar such that their security policies are strategic complements, then $dG^i_T/d\tau^i > 0$ and $dG^j_T/d\tau^i > 0$ hold. It also follows that $dG^i_T/d\tau > 0$ and $dG^j_T/d\tau > 0$ for $\tau^i = \tau^j = \tau$.

(b) Now suppose the international distribution of secure factor endowments is uneven, with country i having sufficiently greater secure endowments of both capital and labor to imply $V^i_{G^iG^j} > 0$ while $V^j_{G^jG^i} < 0$. Based on the preceding analysis, it follows that $dG^i_T/d\tau^i > 0$, as illustrated for country $i = 1$ in Fig. B.3(a). Furthermore, numerical simulations indicate that $dG^j_T/d\tau^i > 0$ under most situations, which include the possible presence of asymmetric endowments. Nevertheless, for sufficiently uneven distributions of secure endowments across countries, $dG^j_T/d\tau^i < 0$ can hold, as illustrated in Fig. B.3(b) for $j = 2$. This figure also illustrates that G^j responds non-monotonically to changes in trade costs τ^i. Moreover, it is possible for $G^j_T > G^j_A$ to hold at some trade-cost levels, including trade costs that are sufficiently close to 1 (free trade), as discussed earlier in the paper. This possibility can be seen in the figure by comparing the curve labeled as $\tau^2 = \infty$ (which corresponds to autarky, since $\alpha = \infty$) with the other ones.

(c) Nonetheless, under most circumstances, globalization (i.e., $\tau^i \downarrow$ and/or $\tau^j \downarrow$) induces both adversaries to reduce their equilibrium arming. Exhaustive numerical analysis also establishes that $d(G^i_T + G^j_T)/d\tau^i > 0$ under all circumstances (even when $dG^j_T/d\tau^i < 0$, as expected since country j's sufficiently small size implies this effect is relatively small).

Turning to payoffs, inspection of the welfare decomposition shown in (B.16) reveals that the direct effect of globalization on either country's payoff is positive. Since the smaller country's rival reduces its arming, the strategic payoff effect reinforces that direct effect. Hence, globalization is always welfare enhancing for smaller countries. For the larger country, whose rival arms by more as with globalization, the strategic effect moves in the opposite direction. However, exhaustive numerical analysis confirms that, even in this case, the direct (and positive) effect dominates. Thus, we have $dV^i_T/d\tau^i < 0$ and $dV^j_T/d\tau^i < 0$, which also suggests that $V^i_T > V^i_A$ for $i = 1, 2$.

of *residual* secure capital to *residual* secure labor, k^i_Z shown in (7), after arming choices have been made.

[20] That is, $\text{sign}\{V^i_{G^i\tau^i}\} = -\ \text{sign}\{d\rho^i/d\tau^i\} = -\ \text{sign}\{d(\gamma^i/\Delta)/d\tau^i\} > 0$ and similarly for the sign of $V^j_{G^j\tau^i}$.

Tariffs. Now we consider $s = t^i$, assuming no non-tariff trade costs: $\tau^i = \tau^j = 1$. In this case, country i's FOC with respect to arming at an interior solution requires $\frac{\partial V^i/\partial G^i}{V^i}|_{\tau^i=1} = 0$. Analyzing the effects of tariffs on equilibrium arming and payoffs is considerably more complex than analyzing the effects of non-tariff trade barriers. Specifically, because tariffs generate revenues, signing the direct effects of tariffs on the marginal payoffs to arming $V^i_{G^i t^i}$ and $V^j_{G^j t^i}$ becomes very difficult.[21] Matters are further complicated by the dependence of the parameter ρ^i on the signs of the cross partials $V^i_{G^i G^j}$, which are themselves difficult to identify since changes in security policies also affect prices. Finally, establishing quasi-concavity of V^i in G^i and uniqueness of equilibrium under trade is also difficult. Even so, we can use numerical methods to calculate (the unique) equilibrium under trade and its dependence on tariffs. Highlighting our more interesting findings, we have:

(a) When the distribution of secure resources is severely uneven, the larger country i's security policy G^i can be non-monotonic in its own tariff t^i. In particular, as illustrated in Fig. B.4(a) for $i = 1$, whether $t^2 = 1$, $t^2 = 2$, or $t^2 = t^1$, $dG^1_T/dt^2 < 0$ at low t^1 levels, but $dG^1_T/dt^1 > 0$ at high t^1 levels.[22] This finding suggests that a large country's (i) trade policy (t^i) can serve as a substitute for its security policy (G^i_T) at low tariff levels but as a complement at high tariff levels. By contrast, $dG^j_T/dt^i > 0$ at low values of t^i, whereas $dG^j_T/dt^i < 0$ at high t^i values, as illustrated in Fig. B.4(b) for $j = 2$. Thus, a small country (j) responds to protectionist moves by its relatively large adversary (i) by increasing (reducing) its arming G^j_T at low (high) tariff rates.

(b) The findings above do change with alternative endowment configurations. In the case of complete symmetry, for example, numerical analysis suggests the emergence non-monotonicities with respect to changes in t^i depends on the value of t^j. In particular, when t^j is not large, more liberal trade policies by country i ($t^i \downarrow$) are accompanied by less aggressive security policies by both countries; $dG^i_T/dt^i > 0$ and $dG^j_T/dt^i > 0$.

(c) Numerical analysis reveals further (not shown) that trade liberalization by a small county i ($t^i \downarrow$) always induces a less aggressive security policy by its larger rival (j): $dG^j_T/dt^i > 0$. But, such a trade policy by country i tends to affect its own security policy non-monotonically and in a way that depends on its rival's (j) trade policy. For example, if $t^j = 1$, then $\lim_{t^i \to 1} dG^i_T/dt^i > 0$, but $dG^i_T/dt^i < 0$ at large t^i values; however, if t^j is large, then $dG^i_T/dt^i < 0$ for all t^i.

[21] From (B.16), we see that $\text{sign}\{V^i_{G^i t^i}\} = -\ \text{sign}\{d\rho^i/dt^i\}$, where ρ^i now depends on t^i directly and indirectly through its dependence on expenditure shares and uncompensated price elasticities of import demands which, in turn, depend on t^i directly and indirectly through internal and external prices. The sign of $V^j_{G^j t^i}$ is simpler to compute since it does not depend directly on t^i. However, it, too, depends on the various elasticities noted above and expenditure shares.

[22] The last case where $t^2 = t^1$ is relevant when countries agree to sign reciprocal trade agreements that involve equal tariff concessions to each other.

Our numerical analysis also reveals a number of interesting tendencies for equilibrium arming under various non-cooperative equilibria. Specifically, we compare arming under (i) autarky ($t^j = \infty$), (ii) free trade ($t^i = t^j = 1$) and (iii) "generalized war" ($t^i = t^i_N$) which we identify with the Nash equilibrium in tariff and security policies).[23] Letting the subscripts "A," "F," and "W" respectively denote equilibrium values under autarky, free trade and generalized war, we observe the following tendences:

(a) Under complete symmetry (and most circumstances):
$G^i_F < G^i_W < G^i_A$ for $i = 1, 2$.

(b) When country i (j) is extremely large (small):
$G^i_W < G^i_F < G^i_A$, while $G^j_A < G^j_F < G^j_W$ for $i \neq j$.

As expected, under complete symmetry (and more generally), autarky induces both countries to produce more guns as compared with the other equilibria. Free trade leads to relatively less arming by each. However, for extremely uneven distributions in factor endowments, matters change, as illustrated in Figs. B.4(a) and B.4(b).[24] While the large country ($i = 1$) continues to produce more arms under autarky, it tends to arm by less under a generalized war than under free trade. We view this latter result as reflecting the added flexibility (and muscle) afforded by generalized war for this country to manipulate its TOT more effectively via its trade policy t^1. The relatively smaller country ($j = 2$) tends to arm by more under a generalized war than under free trade for exactly the opposite reasons. The comparison of aggregate arming across these regimes is determined by the relatively larger economy's guns.

With (B.16), let us examine the welfare implications of tariffs. Focusing on dV^i_T/dt^i, first note that the direct effect of an increase in t^i on V^i_T is positive (negative) if $t^i < t^i_o$ ($t^i > t^i_o$). We also need to examine the indirect effects. But, by the envelope theorem, only the effect of t^i on G^j is relevant. Keeping in mind that $\partial V^i_T/\partial G^j < 0$, the nature of this strategic effect depends on the initial level of t^i. Fig. B.5(a) shows, for $i = 1$, that $\lim_{t^i \to 1} \partial V^i_T/\partial t^i > 0$ and $\lim_{t^i \to \infty} \partial V^i_T/\partial t^i < 0$. The same figure also illustrates that it is possible for country 1 to use its trade policy to improve its payoff beyond the level under free trade V^1_F.

Turning to the effects of t^i on V^j_T, we can infer from (B.16) that the direct effect is negative and that indirect effect due a change in G^i_T depends on the initial level of t^i. In

[23]The necessary FOCs for an interior equilibrium of the simultaneous-move game, which to the best of our knowledge has not been studied before, are: $V^i_{G^i} = 0$ and $V^i_{t^i} = 0$ ($i = 1, 2$), which involve four equations in four unknowns. $V^i_{G^i} = 0$ is familiar from our earlier analysis. $V^i_{t^i} = 0$ requires each country i to use its tariff to balance at the margin its welfare gain due to a TOT improvement against its welfare loss due to an adverse volume-of-trade effect. As noted earlier, $t^i = t^i_o$ where $t^i_o = \varepsilon^j/(\varepsilon^j - 1)$, $j \neq i = 1, 2$.

[24]That the quantities of guns associated with these equilibria are shown as being independent of t^1 in the figures is due to the fact that both guns and tariffs are endogenous.

the case depicted in Figs. B.4 and B.5 for country $j = 2$, the indirect effect is positive for small t^i values and negative for large t^i values. Still, it appears that the direct effect dominates, so that $dV_T^2/dt^1 < 0$. Because in all other cases the strategic effect is negative, we also have $dV_T^2/dt^1 < 0$.

Finally, we compare the payoffs under the various equilibria identified above (i.e., autarky, generalized war, and free trade) and also throw into the mix the payoffs often considered in the literature that presumes no resource insecurity and thus no arming. Specifically, we consider free-trade under "Nirvana" (or no arms) indicated with the subscript "FN" and a trade war under "Nirvana" (again no arms) indicated with the subscript "WN." The payoff rankings depend on the distribution of secure endowments as follows:

(a) Under complete symmetry (and most circumstances):
$V_A^i < V_W^i < V_F^i < V_{WN}^i < V_{FN}^i$ for $i = 1, 2$.

(b) When country i (j) is extremely large (small):
$V_{FN}^i < V_{WN}^i < V_A^i < V_F^i < V_W^i$ and $V_A^j < V_W^j < V_F^j < V_{WN}^j < V_{FN}^j$ for $i \neq j$.

As is well-known from standard theory, under complete symmetry and secure property, a trade war (WN) has the features of a prisoner's dilemma problem relative to free trade (FN), such that $V_{WN}^i < V_{FN}^i$. What is not known is that we also have $V_A^i < V_W^i < V_F^i$ in this case. Interestingly, this ranking of payoffs is preserved for the smaller country when the distribution of secure endowments is sharply uneven. By contrast, the payoff rankings for larger country change in this case. We see that it prefers a generalized war (W) over a non-cooperative equilibrium in security policies coupled with free trade (F). While this result might seem surprising, it is consistent with the finding of Syropoulos (2002) that, in the absence of insecure property, a sufficiently large country "wins" a tariff war over its smaller trading partner. But, what is perhaps striking is that the extremely large country prefers all of these equilibria to the ones under Nirvana (WN and FN).

B.2.4 Trade Costs between Friends

In this section, first we show how the model of three countries must be modified to incorporate non-tariff trade costs and conduct some preliminary analysis. After presenting our proof to Proposition 7, we then provide some details concerning our numerical analysis.

Introducing non-tariff trade barriers. For our analysis of the three country case with non-tariff trade costs, we select good 1 to be the numeraire and work with the relative price p^h in country $h = 1, 2, 3$ of good 2, which is the good in which country 3 enjoys a comparative advantage. Optimization in consumption gives the standard demand functions $D_2^i = \gamma_2^i Z^i / p^i$ for $i = 1, 2$ and $D_2^3 = \gamma_2^3 Z^3$, where $\gamma_2^i = (p^i)^{1-\sigma}/(1 + (p^i)^{1-\sigma})$ for $i = 1, 2, 3$. Then, maintaining the assumption that each country specializes completely in the good in which enjoys a comparative advantage, we use these demand functions to write the world market-clearing condition for good 2 as

$$\tau^1 D_2^1 + \tau^2 D_2^2 + D_2^3 - Z^3 = 0, \tag{B.20}$$

where (as defined in the text) τ^i reflects country i's (iceberg) cost of importing good 2 from the friendly country 3; country 3's cost of importing good 1 from country i is τ_i^3 ($i = 1, 2$).

Now let q_1^i and q_2^i denote the prices at which the goods 1 and 2 are exchanged between countries i and 3. Also let $q^i = q_2^i/q_1^i$ denote the relative external price of good 2 for country $i = 1, 2$. For each country i, the no-arbitrage condition requires $p^i = p_2^i/p_1^i = \tau^i q_2^i/q_1^i = \tau^i q^i$. From country 3's perspective, we have $p^3 = p_2^3/p_1^3 = q_2^i/(\tau_i^3 q_1^i) = q^i/\tau_i^3$, which implies $q^i = \tau_i^3 p^3$. Substituting this value of q^i into $p^i = \tau^i q^i$ gives $p^i = \tau^i \tau_i^3 p^3$, which is the no-arbitrage condition for adversarial country i ($= 1, 2$), expressed in terms of p^3.[25]

Totally differentiating (B.20), one can find

$$\widehat{p}^3 = \frac{1}{\Delta}\left[\nu^i \widehat{Z}^i + \nu^j \widehat{Z}^j + \nu^i \left(\varepsilon^i - 1\right)\widehat{\tau}^i + \nu^j \left(\varepsilon^j - 1\right)\widehat{\tau}^j - \nu^i \varepsilon^i \widehat{\tau}_i^3 - \nu^j \varepsilon^j \widehat{\tau}_j^3\right], \tag{B.21a}$$

for $i \neq j = 1, 2$, where

$$\Delta = \nu^1 \varepsilon^1 + \nu^2 \varepsilon^2 + \varepsilon^3 - 1 \tag{B.21b}$$

$$\varepsilon^i = 1 + (\sigma - 1)\left(1 - \gamma_2^i\right), \quad i = 1, 2 \tag{B.21c}$$

$$\varepsilon^3 = 1 + (\sigma - 1)\gamma_2^3 \tag{B.21d}$$

$$\nu^i = \frac{\gamma_2^i Z^i / \left(\tau_i^3 p^3\right)}{\gamma_2^1 Z^1 / \left(\tau_1^3 p^3\right) + \gamma_2^2 Z^2 / \left(\tau_2^3 p^3\right)}, \quad i = 1, 2, \tag{B.21e}$$

and where, as before, ν^i represents country i's import share (and, under complete specialization, its demand share) of good 2.[26] Assuming $\sigma > 1$, we have $\varepsilon^i > 1$ for $i = 1, 2, 3$ and once again $\Delta > 1$. In fact, as one can verify, $\Delta > \sigma$.[27]

Now differentiate country i's indirect utility function V_T^i, as shown in (9) with $p^i = \tau^i \tau_i^3 p^3$, with respect to G^i. Recognizing the dependence of the market clearing price p^3 on guns through Z^i for $i = 1, 2$ as shown in (B.21a) gives country country i's ($\neq j = 1, 2$) FOC

[25] Observe that $p^3 = q^i/\tau_i^3$ for $i = 1, 2$, as domestic prices in 3 must be the same regardless of whether good 1 is imported from 1 or 2. Of course, if $\tau_i^3 = \tau_j^3$ for $i \neq j = 1, 2$, then $q^i = q^j$.

[26] Note that, if $\tau_1^3 = \tau_2^3$ and $\tau^1 = \tau^2$, then $\gamma_2^1 = \gamma_2^2$, and thus $\nu^i = Z^i / \left(Z^1 + Z^2\right)$ for $i = 1, 2$, which is the special case of global free trade we studied in the text.

[27] Specifically, combine (B.21b), (B.21c) and (B.21d), to write $\Delta = 1 + (\sigma - 1)\left[\gamma_2^3 + \nu^i \gamma_1^i + \nu^j \gamma_1^j\right]$ as

$$\Delta = \sigma + (\sigma - 1)[-\gamma_1^3 + \nu^i \gamma_1^i + \nu^j \gamma_1^j] = \sigma + (\sigma - 1)[\nu^i(\gamma_1^i - \gamma_1^3) + \nu^j(\gamma_1^j - \gamma_1^3)].$$

Because consumers in country 3 face a higher relative price for their imports of good 1 than consumers in country i ($= 1, 2$) for the same product, $\gamma_1^i - \gamma_1^3 > 0$ ($i = 1, 2$) holds. Thus, $\sigma > 1$ implies $\Delta > \sigma$.

for its optimizing choice $G^i > 0$ in the presence of trade costs:

$$\frac{dV_T^i/dG^i}{V_T^i} = \left(1 - \frac{\nu^i\gamma_2^i}{\Delta}\right)\frac{dZ^i/dG^i}{Z^i} - \left(\frac{\nu^j\gamma_2^i}{\Delta}\right)\frac{dZ^j/dG^i}{Z^j} = 0. \tag{B.22}$$

Since $\Delta > 1 > \nu^i\gamma_2^i$ and $dZ^j/dG^i < 0$ from (10b), this FOC reveals that $dZ^i/dG^i < 0$ at an interior optimum, which we assume is unique.

Proof of Proposition 7. To characterize the effect of trade costs under the assumption that countries $i = 1, 2$ have identical initial endowments of capital and labor, suppose that $G^i = G^j$ and that the value G^i at that point uniquely satisfies the FOC in (B.22). Assuming $Z^i = Z^j$, we have $dZ^j/dG^j = dZ^i/dG^i$, $dZ^j/dG^i = dZ^i/dG^j$, and

$$\nu^i = \frac{\gamma_2^i/\tau_i^3}{\gamma_2^1/\tau_1^3 + \gamma_2^2/\tau_2^3}, \quad i = 1, 2. \tag{B.23}$$

Next, we calculate the marginal effect of an increase in G^j on V_T^j at that point:

$$\left.\frac{dV_T^j/dG^j}{V_T^j}\right|_{G^j=G^i} = \left[\frac{-\frac{dZ^i/dG^j}{Z^i}}{\Delta\left(1 - \frac{\nu^i\gamma_2^i}{\Delta}\right)}\right]\left[\frac{\gamma_2^i\gamma_2^j}{\gamma_2^1/\tau_1^3 + \gamma_2^2/\tau_2^3}\right]E^j, \tag{B.24a}$$

where

$$E^j \equiv \frac{1}{\tau_i^3}\left(1 - \frac{\gamma_2^i}{\Delta}\right) - \frac{1}{\tau_j^3}\left(1 - \frac{\gamma_2^j}{\Delta}\right). \tag{B.24b}$$

When $\tau^i = \tau^j$ and thus $\gamma_2^i = \gamma_2^j$, our assumption that $\tau_i^3 = \tau_j^3$ implies $E^j = 0$. The unique equilibrium, then, has $G_T^j = G_T^i$. For $\tau^i \neq \tau^j$, the sign of the expression in (B.24a) tells us how G^j compares with G^i; since $dZ^i/dG^j < 0$, it is given by $\text{sign}\{E^j|_{\tau_i^3=\tau_j^3}\}$. It is now easy to show that $\tau^i \neq \tau^j$ implies $\text{sign}\{E^j|_{\tau_i^3=\tau_j^3}\} = \text{sign}\{\gamma_2^j - \gamma_2^i\}$. Since $p^i = \tau^i\tau_i^3 p^3$ for $i = 1, 2$, we have $p^i \gtreqless p^j$ as $\tau^i \gtreqless \tau^j$ for $i \neq j = 1, 2$. Thus, $\text{sign}\{\partial V_T^j/\partial G^j|_{G^j=G^i,\tau_i^3=\tau_j^3}\} = \text{sign}\{\tau^i - \tau^j\}$. Therefore, $\tau^i \lesseqgtr \tau^j$ implies $G_T^i \gtreqless G_T^j$ as claimed. $\quad ||$

Based on the expression for E^j in (B.24b), we can also explore what happens under symmetric bilateral trade costs, where $\tau_i^3 = \tau^i$ for $i = 1, 2$. In this case, we rewrite E^j as

$$E^j\big|_{\tau_i^3=\tau^i} \equiv \frac{1}{\tau^i}\left(1 - \frac{\gamma_2^i}{\Delta}\right) - \frac{1}{\tau^j}\left(1 - \frac{\gamma_2^j}{\Delta}\right) \tag{B.25}$$

We can also rewrite country i's expenditure share on good 2 as $\gamma_2^i = [1 + [(\tau^i)^2 p^3]^{\sigma-1}]^{-1}$. Then, to compare the first and the second terms in E for values of $\tau^i \neq \tau^j$, it suffices to study the dependence of the first term on τ^i. Letting $x \equiv [(\tau^i)^2 p^3]^{\sigma-1}$, differentiation of the first term in (B.25) with respect to τ^i gives

$$
\begin{aligned}
d\left[\frac{1}{\tau^i}\left(1-\frac{\gamma_2^i}{\Delta}\right)\right]/d\tau^i &= \frac{1+x+2\,(\sigma-1)\,x-\Delta\,(1+x)^2}{\Delta\,(\tau^i)^2\,(1+x)^2} \\
&< -\frac{x(1+x)+(\Delta-1)(1+x^2)}{\Delta\,(\tau^i)^2\,(1+x)^2} < 0,
\end{aligned}
$$

where the first inequality in the second line can be confirmed first by recalling $\Delta > \sigma$ (> 1) such that $2x\sigma < 2x\Delta$. Then, after algebraic manipulation and rearrangement, we arrive at the expression in the second line, which is clearly negative. The difference in results where $\tau^i = \tau_i^3$ relative to the case considered in Proposition 7 is due to the fact that differences in $\tau^i = \tau_i^3$ across i $(= 1, 2)$ generate effects not only on expenditure shares, but also on the import shares ν^i shown in (B.23) that have an opposite influence on country j's TOT channel and that dominate the former effect on expenditure shares. In any case, having shown that the first term in the RHS of (B.25) is decreasing in τ^i and, by analogy, the second term is decreasing in τ^j, we have established that $\text{sign}\{\partial V_T^j/\partial G^j|_{G^j=G^i,\tau_i^3=\tau^i}\} = \text{sign}\{E|_{\tau_i^3=\tau^i}\} = -\text{sign}\{\tau^i - \tau^j\}$. Thus, as claimed in footnote 53 of the text, $\tau^i \lesseqgtr \tau^j$ implies $G_T^i \lesseqgtr G_T^j$ when trade costs between trading partners are symmetric.

Numerical Analysis of Trade between Friends. The numerical results presented in Section 5 are based on the same assumptions regarding technologies used in the two-country case. In particular, we assume $\sigma > 1$ and that guns are produced by both countries with labor only and on a one-to-one basis, implying $Z^i = \left(K^i + \phi^i K_0\right)^{1-\theta}\left(L^i - G^i\right)^{\theta}$, where $\theta \in (0, 1)$.

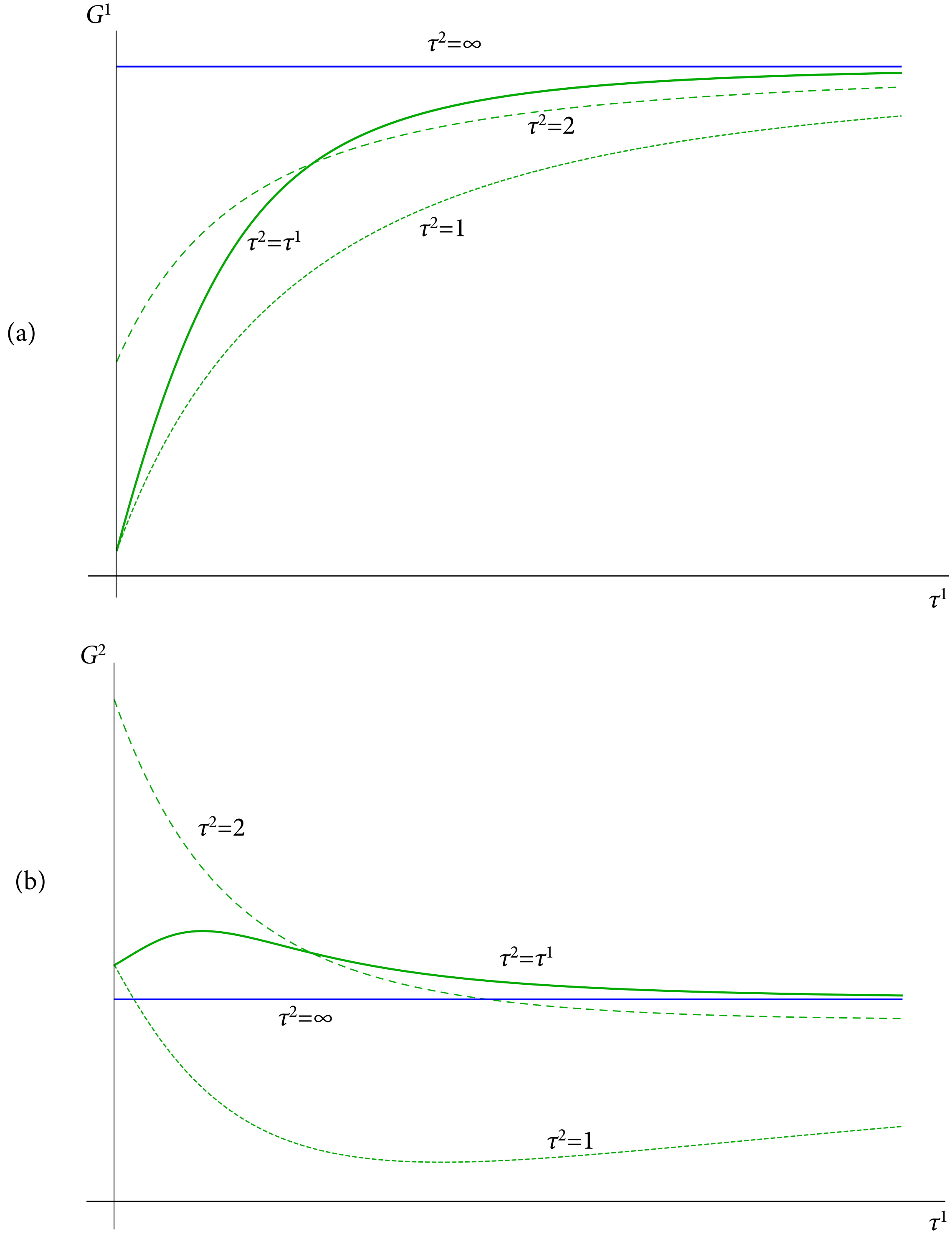

Figure B.3: The Dependence of Arming on Non-Tariff Trade Costs

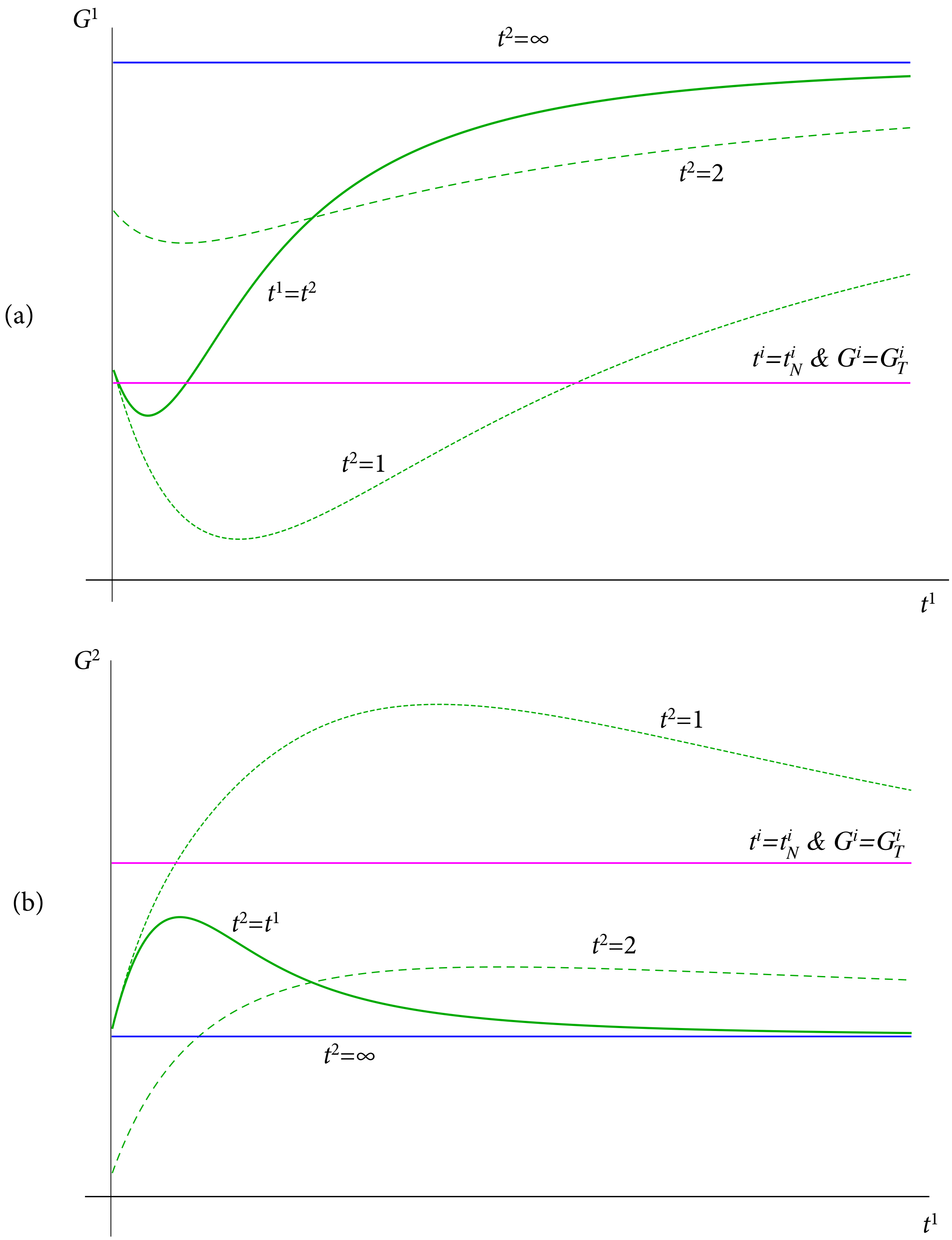

Figure B.4: The Dependence of Arming on Tariffs

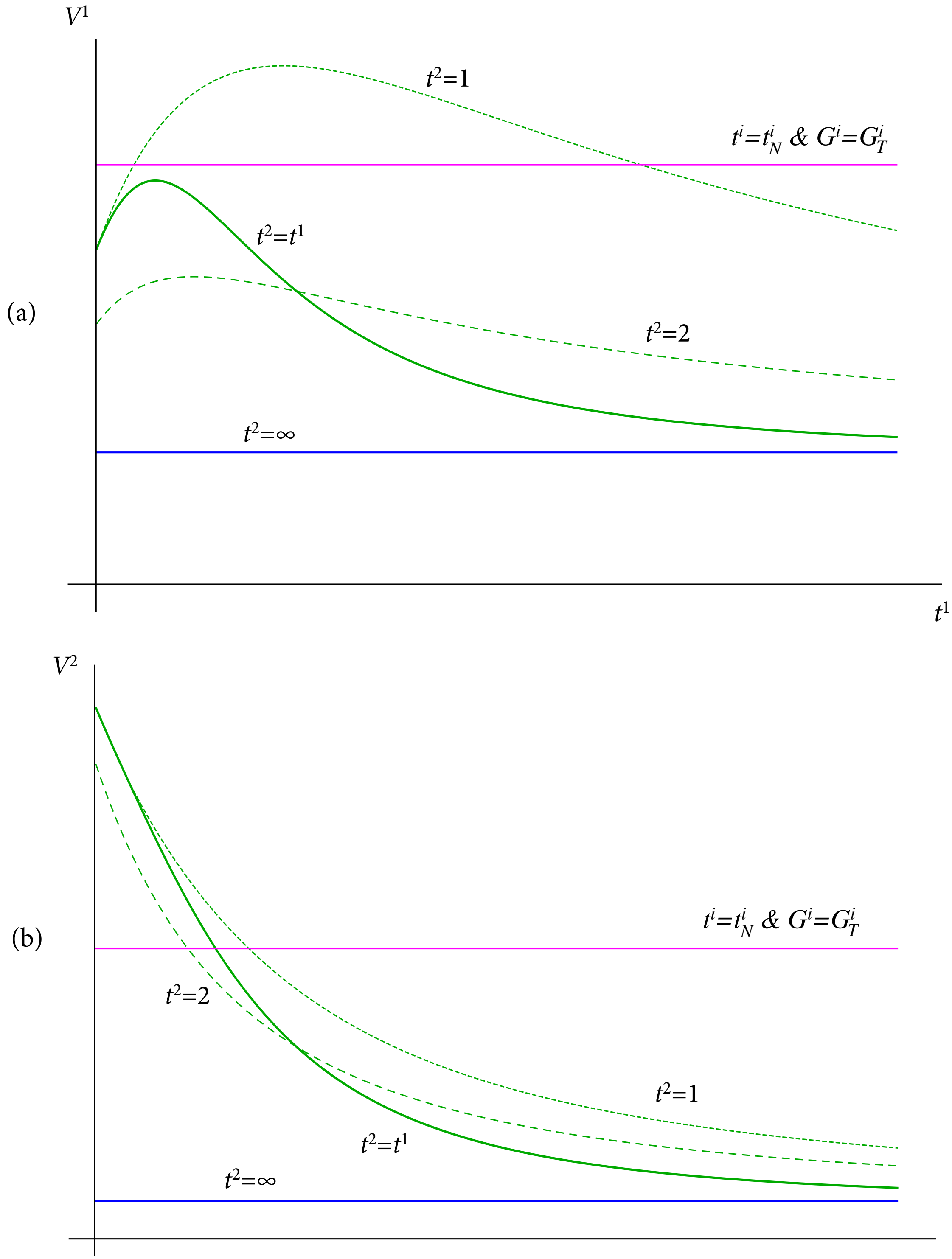

Figure B.5: The Dependence of Payoffs on Tariffs

B.3 Additional Details Underlying the Empirical Analysis

The first section of this part of the supplementary appendix (B.3.1) offers some details regarding the estimating sample. The second section (B.3.2) describes how we obtain bilateral trade costs from a structural gravity model. The third section (B.3.3) presents two alternative procedures to aggregate bilateral trade costs to the country level. Finally, the last section (B.3.4) discusses three robustness experiments.

B.3.1 Country Coverage and Rivalries

The countries included in our sample are listed Table B.1.[28] Table B.2 reports summary statistics for the main variables used in our analysis. Finally, Table B.3 lists the pair-year rivalries from Thompson's (2011) dataset that appear in our estimating sample. As Table B.3 shows, Thompson's indicator variable identifies, within our sample, rivalries between 12 pairs of countries with different durations. Accounting for the fact that the rivalries apply in each direction of trade flows and a number of missing observations on military spending, the total number of observations for rivalries used to estimate the impact of trade costs on military spending in our model is 218 in each of the specifications in Table 1 of the main text.

B.3.2 Obtaining Bilateral Trade Costs

In this section we construct and compare two alternative measures of bilateral trade costs: "estimated" trade costs and "exact-match" trade costs. Both measures are based on the following specification for trade flows from country i to country j in period t, $X_{ij,t}$:

$$X_{ij,t} = \exp[\chi_{i,t} + \varphi_{j,t} + \upsilon_{ij} + \beta_1 FTA_{ij,t} + \beta_2 WTO\_GATT_{ij,t} + \sum\nolimits_t \beta_t BRDR_{ij,t}] \times \epsilon_{ij,t}. \quad \text{(B.26)}$$

This is the econometric version of the structural gravity equation, which is representative of a very wide class of theoretical trade models, as demonstrated by Arkolakis et al. (2012) and summarized in Head and Mayer (2014) and Costinot and Rodriguez-Clare (2014). Based on our comparison of the two trade cost measures, we will choose one for our main analysis.

To ensure our constructed measures are sound, we implement the latest developments in the related literature. First, we follow Santos Silva and Tenreyro (2006) in the use of the Poisson Pseudo-Maximum-Likelihood (PPML) estimator to account for the presence of heteroskedasticity and zeros in trade data. Second, we use time-varying directional (i.e., exporter-time and importer-time) fixed effects ($\chi_{i,t}$ and $\varphi_{j,t}$, respectively) to account for the unobservable multilateral resistances, as motivated by the seminal work of Anderson and

[28] The original dataset of Baier et al. (2019) includes 68 countries. The SIPRI database includes data on military expenditure for all countries except Macau. Thus, our main estimating sample covers 67 countries. It should also be noted that data on military expenditure for a few countries (e.g. Bulgaria, Qatar, and China) are available only for some years. As a result, the number of observations in our sample is 904 (out of a possible 938 observations if the panel were balanced).

van Wincoop (2003). These fixed effects also will absorb national output and expenditure as well as any other observable and unobservable country-specific variables that could impact bilateral trade flows on the importer side or on the exporter side. Third, we employ a flexible country-pair fixed effects approach to control for all (observable and unobservable) time-invariant bilateral trade costs. As demonstrated by Baier and Bergstrand (2007), the use of pair fixed effects, represented by υ_{ij} in (B.26), is an effective method to address the possible endogeneity of any bilateral trade-policy variable. Finally, motivated by the empirical gravity literature, we also include some additional time-varying bilateral covariates, such as (i) $FTA_{ij,t}$, which is a dummy variable equal to 1 when i and j are members of a free trade agreement (FTA) at time t, and zero elsewhere, and (ii) $WTO\_GATT_{ij,t}$, which is a dummy variable equal to 1 when i and j are both members of WTO or GATT at time t, and zero elsewhere. Following Bergstrand et al. (2015), we also include a series of time-varying bilateral border dummies ($\sum_t BRDR_{ij,t}$), which control for common globalization forces.

Our first measure, "estimated" trade costs, uses the estimates of all bilateral variables and fixed effects from equation (B.26) as follows:

$$t_{ij,t}^{1-\sigma} = \exp[\hat{\upsilon}_{ij} + \hat{\beta}_1 FTA_{ij,t} + \hat{\beta}_2 WTO\_GATT_{ij,t} + \sum\nolimits_t \hat{\beta}_t BRDR_{ij,t}]. \tag{B.27}$$

This expression defines the power transform of bilateral trade costs based on structural gravity theory, where $\sigma > 1$ can be interpreted broadly as the trade elasticity (Arkolakis et al., 2012).[29]

The rich fixed effects structure of specification (B.26) (including bilateral fixed effects, time-varying bilateral borders, exporter-time fixed effects, and importer-time fixed effects) supports the assumption of a stochastic error term, $\epsilon_{ij,t}$. However, we cannot rule out the possibility that $\epsilon_{ij,t}$ contains some systematic trade cost information. Anderson et al. (2018) propose a hybrid approach, which uses a model similar to (B.27) to estimate the effects of trade policy, and then adds the error to the trade-cost function in order to fit the trade flows data perfectly. Specifically, "exact-match" trade costs can be constructed as follows:

$$\tilde{t}_{ij,t}^{1-\sigma} = t_{ij,t}^{1-\sigma} \times \epsilon_{ij,t} = \exp[\hat{\upsilon}_{ij} + \hat{\beta}_1 FTA_{ij,t} + \hat{\beta}_2 WTO\_GATT_{ij,t} + \sum\nolimits_t \hat{\beta}_t BRDR_{ij,t}] \times \epsilon_{ij,t}. \tag{B.28}$$

Below we compare the two constructs obtained from (B.27) and (B.28) to determine whether it is reasonable to treat the error term in (B.26) as stochastic.[30]

Before that, let us briefly discuss the estimates of the bilateral gravity variables obtained from specification (B.26). These estimates are reported in Table B.4. For the effects of

[29] Our notation here $t_{ij,t}$, which aims to capture trade costs generally, should not be confused with our notation above in Section 4.2 and Supplementary Appendix B.2 (where t denotes tariffs).

[30] In the robustness analysis, we also experiment by constructing and using an alternative measure of bilateral trade costs, which relies only on the estimates of the bilateral fixed effects, $\hat{\upsilon}_{ij}$, from (B.26).

FTAs, we obtain a statistically significant estimate of 0.375 (std.err. 0.032), which implies that, *ceteris paribus*, the FTAs that entered into force during the period of investigation promoted trade between their member countries by about 45.5 percent ($100 \times [\exp(0.375) - 1]$). The estimate of the effect of WTO/GATT is also positive and statistically significant, suggesting that WTO and GATT have also been successful in promoting trade among member countries. Finally, consistent with Bergstrand et al. (2015), our estimates of the border-time fixed effects ($\sum_t BRDR_{ij,t}$) reveal that the impact of international borders has fallen over time, i.e., they capture the impact of globalization.[31] Overall, these estimates as plausible and comparable to corresponding estimates from the related literature.[32]

Next, we construct and compare the two measures of bilateral trade costs that are based on (B.26), to choose between them for use in the second-stage of our analysis of military spending. For this discussion, we focus on the last year in our sample, 1999; however, our conclusions hold for each year in the sample. First, using a conventional value of the elasticity of substitution $\hat{\sigma} = 6$ (c.f., Anderson and van Wincoop (2003) and Head and Mayer (2014)), we recover bilateral trade costs in levels ($\widehat{t}_{ij,t}$), which are all positive and greater than one, as suggested by theory.[33] We note that the recovered values of $\widehat{t}_{ij}$ using the "estimated" measure (B.27) vary widely but intuitively across the country pairs. The lowest estimates are for country pairs that are geographically and culturally close and economically integrated (e.g., USA and Canada), while the largest are for geographically remote and economically less developed country pairs (e.g., Ecuador and Nepal).

Second, we compare the "estimated" trade costs from (B.27) with the "exact-match" trade costs indexes from (B.28). The correlation between the two measures for all countries in 1999 is 0.987. In addition, on a country by country basis, the correlations are positive, varying between 0.40 and 0.99, and are quite strong in a very large majority of cases. To get a better sense of the fit between "estimated" and "exact-match" bilateral trade costs, we divide the countries into 3 groups. The first group includes the seven countries in our sample (Malawi, Jordan, Myanmar, Kuwait, Malta, Nigeria, and Panama) having correlations between the two sets of trade costs below 0.7. Malawi and Jordan are the only two countries with a correlation of less than 0.5. Fig. B.6 illustrates the fit between the "estimated" and the "exact-match" trade costs for three representative countries (Panama, Myanmar, and Malawi) in this group. In this figure (and the two that follow), the trade-cost

[31]Note that the use of pair fixed effects does not allow the inclusion of border variables for all years, and we need to drop one time-varying border variable. We chose the border for 1986. This means that the estimates of all other border dummies should be interpreted as deviations from the border in 1986. Thus, the positive and increasing estimates that we obtain indeed imply decreasing international border effects.

[32]See Head and Mayer's (2014) excellent meta analysis for a benchmark set of gravity estimates.

[33]The structural gravity model can only identify relative trade costs and, with the pair-fixed effects in our estimations, we have to impose N (67) normalizations. A natural choice is to set all intra-national trade costs to one ($t_{ii} = 1, \forall i$). Thus, we estimate international trade costs relative to intra-national trade costs.

observations for a given country i against all other countries $j \neq i$ are ordered (along the horizontal axis) from smallest to largest according the "exact-match" measure. Fig. B.6 and the corresponding correlations reveal four patterns: (i) the worst correlations are for poor nations, which are suspects of measurement errors in the reported data; (ii) the fit is better for trade costs with partners that have relatively low trade costs; (iii) the dispersion is wider for larger trade costs; and (iv) "estimated" trade costs somewhat under-predict larger "exact-match" trade costs.

The second group includes the nine countries with correlations ranging from 0.7 to 0.9. Fig. B.7, which depicts the fit between the "estimated" and the "exact-match" trade costs for three representative countries in this group (Cyprus, Bulgaria, and Iran), reveals a relatively good fit, with a bit wider dispersion corresponding to higher bilateral trade costs. The last group includes the remaining countries (roughly $\frac{3}{4}$ of those in our sample), all with correlations greater than 0.9; for most of them, it is larger than 0.95. Fig. B.8 illustrates the fit between the "estimated" and the "exact-match" trade costs for three representative countries in this group, Austria, France, and the United States.

Finally, we note that we could not obtain estimates of the pair fixed effects (due to zero trade flows throughout the whole period of investigation) for ten pairs of countries in our sample. Only one of those pairs, Israel-Iran, was involved rival countries. To obtain our main results, we treated the corresponding bilateral trade-cost observations (less than 0.4 percent in total) as missing. In addition, we also performed sensitivity analysis, where we set the missing estimates of the pair fixed effects to be equal to the largest (in absolute value) in the sample. Our main results remained robust.

In combination, (i) the good overall fit between "estimated" and "exact-match" trade costs, (ii) the small fraction of trade flows for which "estimated" trade costs do not match their "exact-match" counterparts well, which is concentrated in the case of poorer countries, and (iii) the fact that trade data with less developed countries are noisier, suggest that the error term in specification (B.26) may indeed be stochastic, reflecting noise rather than systematic information. Therefore, the analysis in this section motivates and justifies our choice to use the vector of "estimated" trade costs for the analysis of the determinants of military spending.

B.3.3 Aggregating Bilateral Trade Costs

The next step we need to take to move towards the analysis of the determinants of military spending is to aggregate the bilateral trade costs that we obtained in the previous section to the country level. To this end, we consider two alternative aggregation procedures, both of which are motivated by the structural gravity model of trade. For both, we define and focus on $\tau_{ij,t} \equiv t_{ij,t}^{\sigma-1}$, to preserve the inverse relationship between trade flows and trade costs. In the first aggregation procedure, we construct country-time specific weighted-average trade costs for each exporter in our sample with importer expenditure shares used as weights:

$$\tau_{i,t} = \sum\nolimits_j \tau_{ij,t} \frac{E_{j,t}}{\sum E_{j,t}}. \tag{B.29}$$

This procedure is motivated by structural gravity theory, but it is not perfectly consistent with the structural gravity model, since proper aggregation should also take into account the structural multilateral resistances.[34] In particular, the gravity model implies the following theory-consistent aggregation of bilateral trade costs to the country level:

$$\tau_{i,t} = \sum\nolimits_j \tau_{ij,t} \frac{E_{j,t}/P_{j,t}^{1-\sigma}}{\sum_j E_{j,t}/P_{j,t}^{1-\sigma}}. \tag{B.30}$$

While consistent with gravity theory, the procedure in (B.30) introduces additional endogeneity concerns for our purposes, since the multilateral resistances are general-equilibrium, trade-cost indexes that include national income by construction. Therefore, to obtain our main results, we employ equation (B.29) to aggregate bilateral trade costs up to the country-time level. However, in the sensitivity analysis presented in the next section, we also experiment with and demonstrate the robustness of our results when bilateral trade costs are aggregated according to (B.30). To address potential endogeneity concerns in that case, we also employ an IV estimation strategy following the methods of Feyrer (2009, 2019) described in the main text.

B.3.4 Robustness Analysis

We conclude with three robustness experiments. The first one employs an alternative measure of the underlying bilateral trade-cost vector, based only on the estimates of the bilateral fixed effects from the first stage:

$$t_{ij}^{1-\sigma} = \exp[\hat{\upsilon}_{ij}] \tag{B.31}$$

The motivation for this specification is to mitigate concerns due to the use of potentially endogenous trade policy variables in our definition of trade costs. Our findings appear in Table B.5, where, to ease comparison, we reproduce in column (1) the main IV estimates from column (3) of Table 1 in the main text. For brevity of exposition, we only report estimates based on the new trade costs from our most preferred IV specification. The estimates in column (2) of Table B.5 confirm the robustness of our main findings to the use of the alternative pair-fixed-effects measure of the underlying bilateral trade-cost vector.

In the second experiment, we employ the structural weights according to specification

[34] We refer the reader to Anderson and Neary (2005) and Agnosteva et al. (2014) for details and discussion of consistent aggregation of bilateral trade costs with the gravity model.

(B.30) to aggregate the bilateral trade costs to the country level. As noted before, the introduction of the multilateral resistances (ideal consumer price indexes) could cause additional endogeneity concerns. Therefore, once again, our preferred specification with this covariate is the IV approach. The estimates in column (3) of Table B.5 are very similar to our main findings from column (1) of the same table and, therefore, confirm their robustness.

In the last experiment, we test the robustness of our results by focusing on the Cold War years in our sample, specifically 1986–1991. Once again, the estimates in column (4) of Table B.5 confirm our main findings regarding the relationship between military spending and trade with friends and rivals. In particular, the estimates of the coefficients on our two key trade-cost covariates remain statistically significant, their signs are as predicted by theory, and their magnitudes are comparable to our main findings.

Given the reduced-form nature of our evidence, we also considered one alternative explanation for our main findings. In particular, since more aggressive countries have more far-reaching geopolitical ambitions, they tend to have rivals that are located further away and thus greater trade costs with them. At the same time, they tend to have larger military budgets. To test whether these combined tendencies could explain our findings, we split each of the two key trade cost covariates at their respective means. The results for trade costs with friends and the results for trade costs with rivals were very similar for smaller and for larger trade costs.[35]

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[35] Details are available on request from the authors.

Table B.1: Country Coverage

ISO Code	Country Name	ISO Code	Country Name
ARG	Argentina	KEN	Kenya
AUS	Australia	KOR	Korea, South
AUT	Austria	KWT	Kuwait
BEL	Belgium	LKA	Sri Lanka
BGR	Bulgaria	MAR	Morocco
BOL	Bolivia	MEX	Mexico
BRA	Brazil	MLT*	Malta*
CAN	Canada	MMR	Myanmar
CHE	Switzerland	MUS	Mauritius
CHL	Chile	MWI	Malawi
CHN	China	MYS	Malaysia
CMR	Cameroon	NER	Niger
COL	Colombia	NGA	Nigeria
CRI	Costa Rica	NLD	Netherlands
CYP	Cyprus	NOR	Norway
DEU	Germany	NPL	Nepal
DNK	Denmark	PAN	Panama
ECU	Ecuador	PHL	Philippines
EGY	Egypt, Arab Rep.	POL	Poland
ESP	Spain	PRT	Portugal
FIN	Finland	QAT	Qatar
FRA	France	ROM	Romania
GBR	United Kingdom	SEN	Senegal
GRC	Greece	SGP	Singapore
HUN	Hungary	SWE	Sweden
IDN	Indonesia	THA	Thailand
IND	India	TTO	Trinidad and Tobago
IRL	Ireland	TUN	Tunisia
IRN	Iran	TUR	Turkey
ISL*	Iceland*	TZA	Tanzania
ISR	Israel	URY	Uruguay
ITA	Italy	USA	United States
JOR	Jordan	ZAF	South Africa
JPN	Japan		

Notes: This table reports the ISO 3-letter codes and the names of the countries in our sample. There were no data on institutional quality for two countries in our sample. These countries are Iceland (ISL) and Malta (MLT), and they are marked with '*'.

Table B.2: Summary Statistics

Variable	**Description**	**Mean**	**Std. Dev.**	**Min.**	**Max.**
TRADE	Bilateral Trade Flows (millions USD)	610.147	3812.957	0	167335.845
DIST_CAP	Distance Between Capital Cities (kilometers)	7440.037	4447.814	111.093	19772.336
FTA	Indicator for Free Trade Agreement	0.123	0.328	0	1
WTO_GATT	Indicator for WTO/GATT Membership	0.790	0.407	0	1
MIL_SPEND	Military Spending (millions USD)	9665.361	36108.057	1	306170
GDP	Gross Domestic Product (millions USD)	567386.77	1389170.214	3594.966	12473554
POPULATION	Population (millions)	62.313	174.246	0.244	1262.714
INSTITUTIONS	Institutional Quality Index	4.651	6.658	-10	10
HOSTILE	Hostility Index	0.015	0.058	0	1.138

Notes: This table reports summary statistics for the main variables in our analysis. We describe the key variable for rivalry in more detail in Table B.3.

Table B.3: Pair-Year Combinations for Rivals

ISO1	ISO2	86	87	88	89	90	91	92	93	94	95	96	97	98	99
ARG	CHL	1	1	1	1	1	1	.	.	.	.	.	.	.	.
ARG	GBR	1	1	1	1	1	1	1	1	1	1	1	1	1	1
BOL	CHL	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CHN	IND	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CMR	NGA	1	1	1	1	1	1	1	1	1	1	1	1	1	1
EGY	IRN	1	1	1	1	1	1	1	1	1	1	1	1	1	1
EGY	ISR	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ESP	MAR	1	1	1	1	1	1	.	.	.	.	.	.	.	.
GRC	TUR	1	1	1	1	1	1	1	1	1	1	1	1	1	1
IRN	ISR	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ISR	JOR	1	1	1	1	1	1	1	1	1	.	.	.	.	.
MWI	TZA	1	1	1	1	1	1	1	1	1	.	.	.	.	.

Notes: This table lists the pair-year combinations of rivalries that enter our sample.

Table B.4: Panel PPML Gravity Estimates

	Gravity Estimates
FTA	0.375
	(0.032)**
WTO_GATT	0.142
	(0.061)*
BRDR_1987	0.019
	(0.023)
BRDR_1988	0.100
	(0.023)**
BRDR_1989	0.108
	(0.026)**
BRDR_1990	0.178
	(0.024)**
BRDR_1991	0.191
	(0.023)**
BRDR_1992	0.216
	(0.021)**
BRDR_1993	0.229
	(0.021)**
BRDR_1994	0.299
	(0.021)**
BRDR_1995	0.374
	(0.021)**
BRDR_1996	0.384
	(0.021)**
BRDR_1997	0.478
	(0.023)**
BRDR_1998	0.533
	(0.023)**
BRDR_1999	0.517
	(0.025)**
N	60355

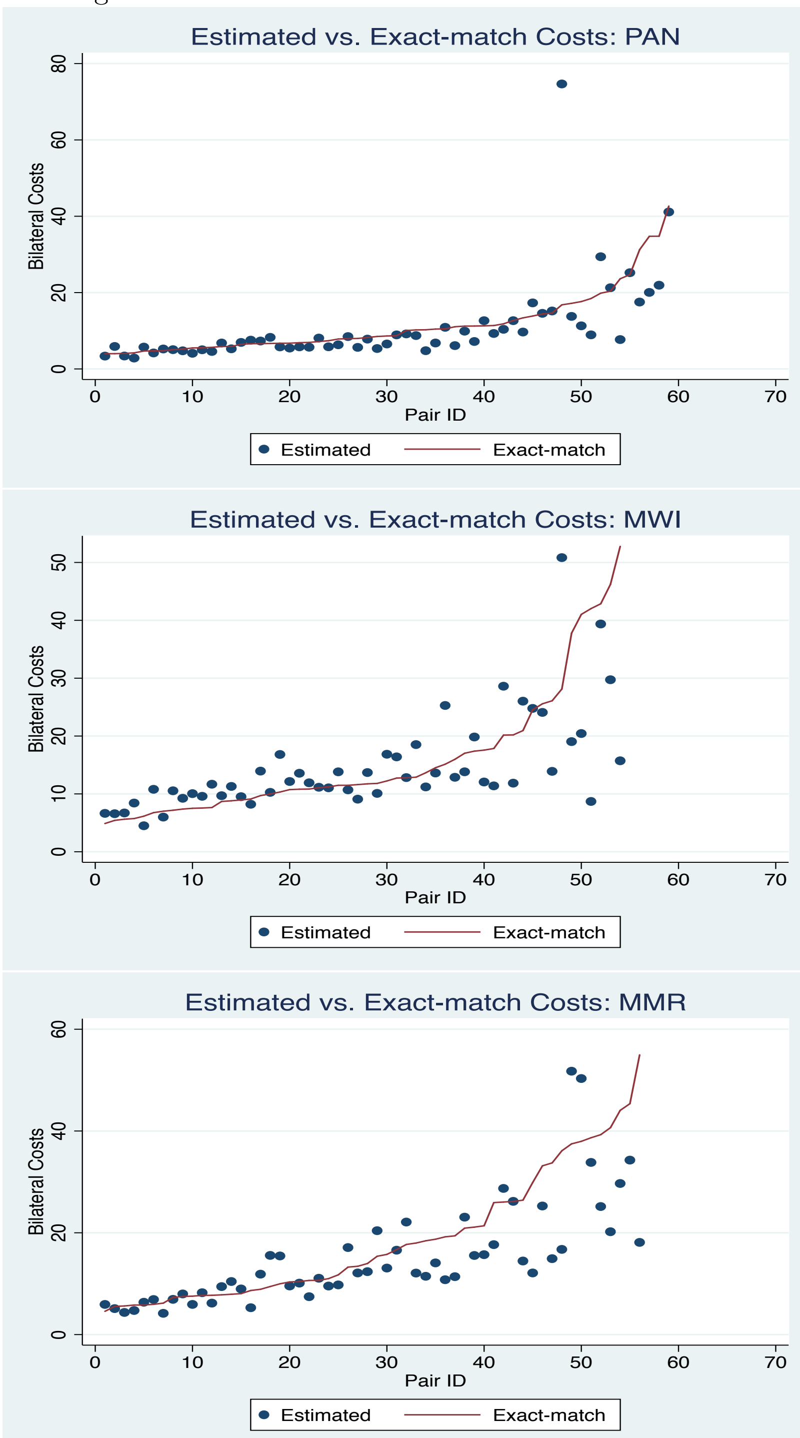
Notes: This table reports results from a panel gravity estimation. The dependent variable is the nominal level of bilateral trade flows, and the estimates are obtained with the PPML estimator. The specification includes exporter-time fixed effects, importer-time fixed effects, and pair fixed effects, whose estimates are omitted for brevity. Standard errors are clustered by country pair and are reported in parentheses. $^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$.

Table B.5: International Trade Costs and Military Spending: Robustness Analysis

	(1) MAIN	(2) PAIR_FEs	(3) STRCTRL	(4) COLD_WAR
TRADE_COSTS_RIVALS	0.214	0.208	0.206	0.261
	(0.082)**	(0.081)*	(0.084)*	(0.126)*
TRADE_COSTS_FRIENDS	-0.226	-0.228	-0.213	-0.322
	(0.065)**	(0.068)**	(0.050)**	(0.095)**
TRADE_COSTS_NR	-0.152	-0.153	-0.166	-0.181
	(0.036)**	(0.038)**	(0.029)**	(0.048)**
GDP	1.347	1.349	1.295	1.383
	(0.068)**	(0.070)**	(0.062)**	(0.111)**
PPLN	-0.507	-0.508	-0.477	-0.567
	(0.044)**	(0.044)**	(0.043)**	(0.077)**
INSTITUTIONS	-0.053	-0.052	-0.051	-0.055
	(0.009)**	(0.009)**	(0.008)**	(0.013)**
HOSTILE	2.525	2.547	2.598	2.381
	(0.441)**	(0.437)**	(0.423)**	(0.502)**
RIVALS	-0.379	-0.376	-0.407	0.052
	(0.416)	(0.431)	(0.459)	(0.618)
RLTV_POWER	0.234	0.223	0.269	0.203
	(0.129)$^{+}$	(0.128)$^{+}$	(0.135)*	(0.218)
N	810	810	810	307
Weak Id χ^2	65.793	62.360	59.905	30.237
Over Id χ^2	1.456	1.431	1.618	1.019

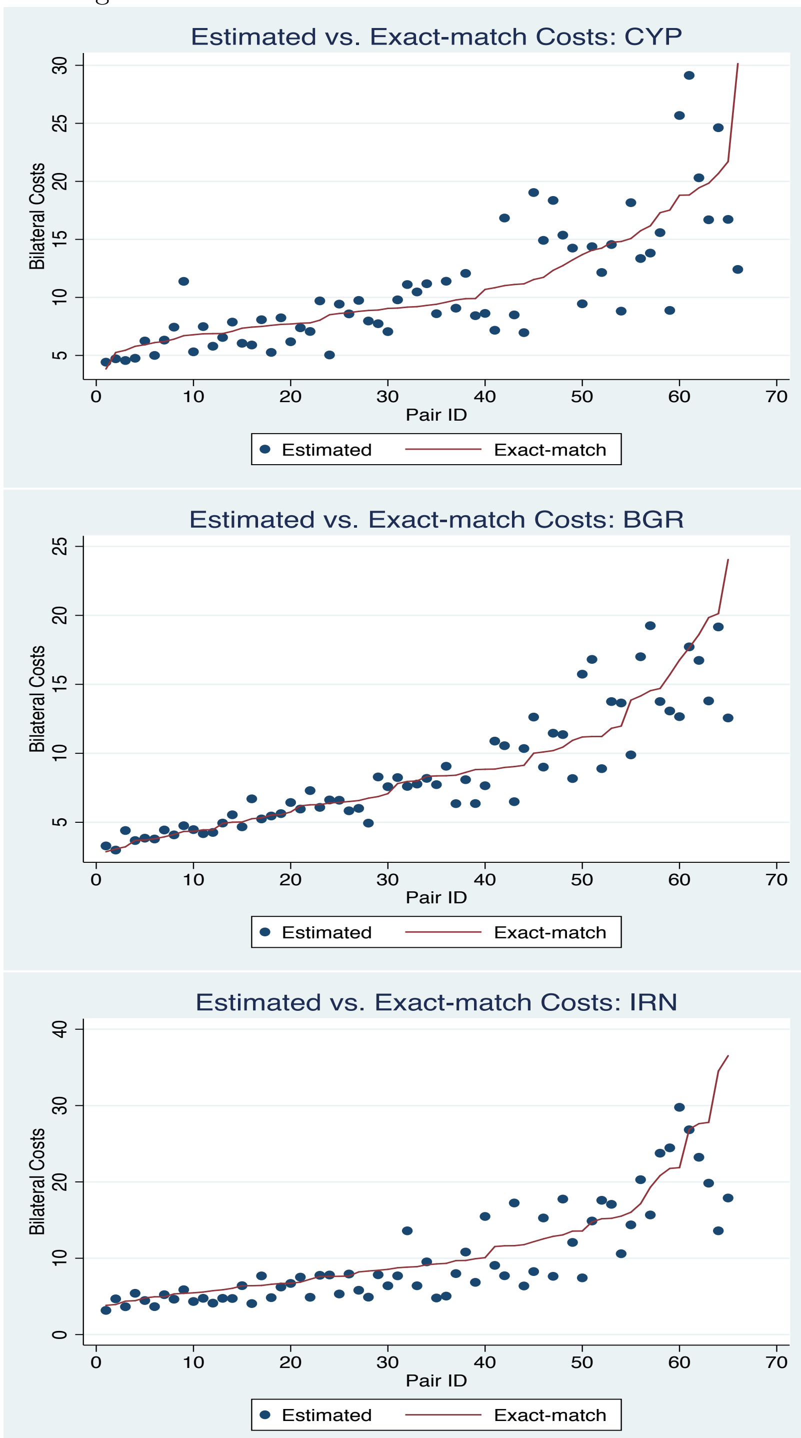
Notes: This table reports the results from three robustness experiments. The dependent variable is always the logarithm of national military spending. All specifications distinguish between the impact of trade costs of countries that have rivals to their rivals (*TRADE_COSTS_RIVALS*) or to their friends (*TRADE_COSTS_FRIENDS*). Each specification includes year fixed effects and all estimates are obtained with an IV estimator. For comparison purposes, the estimates in column (1) are the results from column (3) of Table 1 in the main text. The results in column (2) are based on bilateral trade costs that are constructed only from the estimates of the pair fixed effects using a structural gravity model. Column (3) reports results that are obtained from trade costs that are aggregated with structural gravity weights. Finally, column (4) replicates the main results in column (1) but only for the Cold War years in our sample, i.e. 1986-1991. Robust standard errors are reported in parentheses. $^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$. See the discussion in section B.3.4 of this appendix for further details.

Figure B.6: Estimated vs. Exact-match Trade Costs



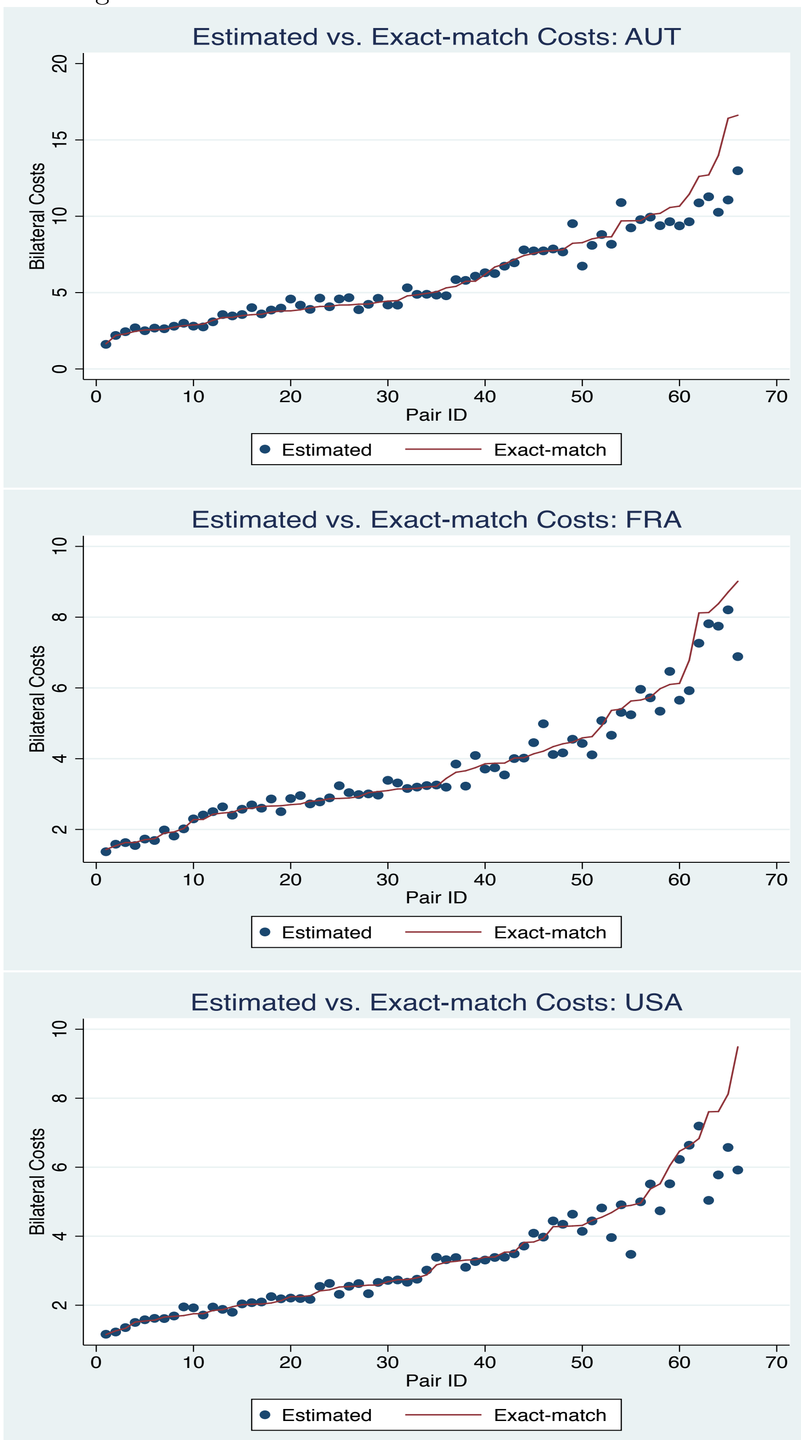
Notes: This figure plots "estimated" vs. "exact-match" trade costs for 1999, which are obtained from gravity specification (B.26), for Panama, Malawi and Myanmar. See text for further details.

Figure B.7: Estimated vs. Exact-match Trade Costs



Notes: This figure plots "estimated" vs. "exact-match" trade costs for 1999, which are obtained from gravity specification (B.26), for Cyprus, Bulgaria, and Iran. See text for further details.

Figure B.8: Estimated vs. Exact-match Trade Costs



Notes: This figure plots "estimated" vs. "exact-match" trade costs for 1999, which are obtained from gravity specification (B.26), for Austria, France, and the United States. See text for further details.